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Current ACI Standards

(As of December, 1951)

Listed below are ACI standards in effect at the time the closing portions of *Proceedings* V. 47 are published in the ACI JOURNAL, Part 2 December 1951:

Manual of Standard Practice for Detailing Reinforced Concrete Structures (ACI 315-51)

Not adaptable to Journal presentation, this is a separate publication of large format bound to lie flat. It presents typical engineering and placing drawings with discussion calling attention to important considerations in designing practice. 55 pages, $3.00; to ACI Members $1.75.

Building Code Requirements for Reinforced Concrete (ACI 318-51)

ACI JOURNAL, Apr. 1951, *Proc.* V. 47, pp. 589-652. In separate covers $0.50; to ACI Members $0.40.

Specifications for Concrete Pavements and Bases (ACI 617-51)

ACI JOURNAL, May 1951, *Proc.* V. 47, pp. 721-744. In separate covers $0.50; to ACI Members $0.40.

Recommended Practice for the Application of Mortar by Pneumatic Pressure (ACI 805-51)

ACI JOURNAL, May 1951, *Proc.* V. 47, pp. 709-720. In separate covers $0.50; to ACI Members $0.40.

Recommended Practice for the Application of Portland Cement Paint to Concrete Surfaces (ACI 616-49)

ACI JOURNAL, Sept. 1949, *Proc.* V. 46, pp. 1-16. In separate covers $0.50; to ACI Members $0.40.

Recommended Practice for Winter Concreting Methods (ACI 604-48)

ACI JOURNAL, Sept. 1948, *Proc.* V. 45, pp. 1-20. In separate covers $0.50; to ACI Members $0.40.

Minimum Standard Requirements for Precast Concrete Floor Units (ACI 711-46)

ACI JOURNAL, Oct. 1946, *Proc.* V. 43, pp. 133-148. In separate covers $0.50; to ACI Members $0.40.

Recommended Practice for the Construction of Concrete Farm Silos (ACI 714-46)

ACI JOURNAL, Oct. 1946, *Proc.* V. 43, pp. 149-164. In separate covers $0.50; to ACI Members $0.40.

Recommended Practice for the Design of Concrete Mixes (ACI 613-44)

ACI JOURNAL, June 1945, *Proc.* V. 41, pp. 651-672. In separate covers $0.50; to ACI Members $0.40. Also available in a Spanish edition: "Normas Practicas Recomendadas Por el ACI Para Proyectar Mezclas de Hormigon (ACI 613-44)." In separate covers $0.50.

Specification for Cast Stone (ACI 704-44)

ACI JOURNAL, June 1945, *Proc.* V. 41, pp. 701-704. In separate prints $0.35.

Recommended Practice for the Use of Metal Supports for Reinforcement (ACI 319-42)

ACI JOURNAL, June 1945, *Proc.* V. 41, pp. 621-624. In separate prints $0.35.

Recommended Practice for Measuring, Mixing and Placing Concrete (ACI 614-42)

ACI JOURNAL, June 1945, *Proc.* V. 41, pp. 625-650. In separate covers $0.50; to ACI Members $0.40.

Valuable information for appraising and advancing design criteria for R/C pavement may be found in survey and maintenance reports of highway departments.

Some Observations on the Use of Reinforcing Steel in Concrete Pavements*

By BENGT F. FRIBERG†

SYNOPSIS

The paper contains a few accumulated observations from a review of literature on the evolution of reinforced concrete pavement designs, with reference to various systems of reinforcement which have seen extended use. Typical structural concrete pavement failures are described. Trends in design and reinforcement practice are shown, with special reference to recent developments. Pertinent findings of a few published pavement surveys are cited, with special attention to the 900-mile Louisiana pavement survey made in 1945. The need for performance information and additional research is stressed.

The use of reinforcing steel in concrete structures is governed by consideration of structural stability and safety; not so in concrete pavements, where safety generally is only remotely related to structural impairment of the pavement. The use—or exclusion—of reinforcing steel in pavements accordingly is guided by other considerations such as economy, preservation of design conditions, maintenance costs and possible extension of serviceable pavement life. As a result, pavement reinforcing practices have varied widely from time to time and service experience is a primary guide to continued development. The following observations from the literature indicate the accumulation of knowledge surrounding the design and use of reinforcing steel in concrete pavements.

EARLY DESIGNS AND REINFORCEMENT USES

Full width construction

Concrete pavements came into use about 50 years ago as an economical replacement for more expensive block-type city pavements. They soon established good service records and gave promise of long life.

*Presented at the ACI 46th annual convention, Chicago, Ill., February 21, 1950. Title No. 47-1 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 22, No. 1, Sept. 1950, *Proceedings* V. 47. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Jan. 1, 1951. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Managing Director, Granco Steel Products Co., Granite City, Ill.

The lack of reliable design information did not seriously affect the pavements for the light loads then common. However, unsightly longitudinal cracking occurred on most pavements over one lane wide and early design practice incorporated relatively heavy reinforcing steel extending from side to side of the pavement to reinforce against these cracks (Fig. 1). The practice of employing substantial amounts of transverse reinforcing steel and building concrete pavements full width prevailed up to about the time of the Bates test road in 1923, and the simultaneous development of the longitudinal center joint.

Special reinforcing systems

During this early period of development, many special reinforcing systems, in the general range of about 1 lb of steel per sq ft of pavement, were proposed or used for individual projects. The practice of employing top and bottom reinforcing layers became common and various built-up reinforcement assemblies were tried and used, without recorded results. Some of these are illustrated in Fig. 1. The use of cold-drawn and welded or woven wire fabric dates back to about 1910, but heavy-edge fabric is not mentioned in the literature until 1924.

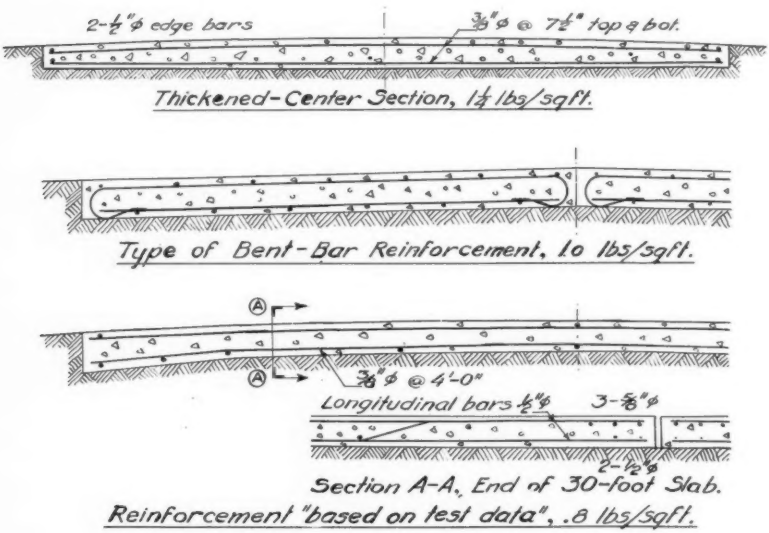


Fig. 1—Some early types of concrete pavement reinforcement

CONCRETE PAVEMENT BEHAVIOR

Evolution of design practice

The literature discloses that there was a fair understanding of the action of a concrete pavement on a soil subgrade as far back as 1915. The first specific mention of steel reinforcing design principles are found in the literature in 1914,[1] as follows:

"The necessity for reinforcement to give structural strength to the slab is evident, regardless of whether the foundation is poor in spots or over the entire area."

"The greater the spacing of joints the greater should be the amount of longitudinal steel."

(For 30-ft expansion joint spacing, 30 lb of steel per 100 sq ft of reinforcement was recommended.)

Generally the early pavements were thicker in the center than along the edges, had no longitudinal joints, and edges were much thinner than now is common. Many of these old pavements continue to serve satisfactorily after 30 years. Some other concrete pavements 25 to 30 years old have not performed so well under heavy traffic. Recent literature discloses numerous cases of salvaging such pavements with temporary 10-year construction at costs of $15,000 or more per mile.[2] This seems a high cost compared to the probable increase in original cost for a more adequate concrete pavement whether the extension in pavement life were obtained by improved subgrade, slab dimensions, material and construction practice, or reinforcement, the performance of which is still to some extent unexplored. It is clear from these observations that no precise and all-inclusive design practice can be expected to determine the use and quantity of steel reinforcement in concrete pavements.

The practice of dividing highway pavements into single-lane width with a thickened outer-edge cross section, and with transverse joints spaced at some specified distance apart has existed in principle unchanged for nearly 30 years, but reinforcing practices for these pavements vary considerably. Generally, no steel is used in highway pavements with transverse joint spacing of 15 ft or less; for longer joint spacings, reinforcement varies from single bars along pavement edges to distributed reinforcement of welded wire fabric, welded or tied bar mats, or expanded metal mesh, in amount of from 0.4 to 0.9 lb per sq ft, the heavier weights generally being used in connection with long transverse joint spacings up to 100 ft (Fig. 2). The relative merits of the various types of distributed reinforcement are still largely matters of conjecture.

It has been assumed from the earliest use of reinforcement that steel in sufficient amount to provide balanced structural reinforcement for the cross section would be impractical to provide, but many early pavement designs show reinforcing steel in relatively large quantity near

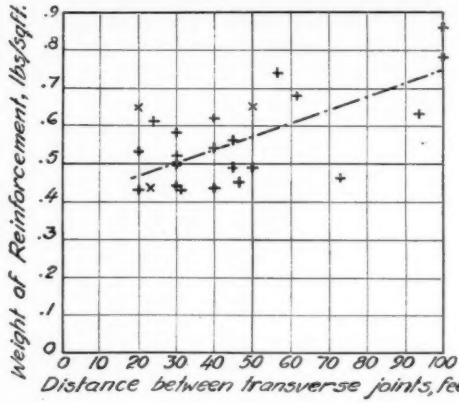


Fig. 2—Weight of distributed reinforcement plotted for transverse joint spacing for highway pavement design standards, 1947

top or bottom of the pavement which in fact added some to the pavement bending resistance. Reinforcement cross-sectional areas of longitudinal steel evolved during the past 30 years are about of 0.1 or 0.2 percent of the pavement cross section. The reinforcement generally is in the upper half of the concrete depth, on the assumption that reinforcement would serve most effectively near the exposed surface.

Reinforcement usually is considered especially in relation to pavement cracking and its anticipated function in a cracked slab. Corners and edges of the pavement generally can be economically designed to sustain the traffic loads by increased thickness or joint design; but transverse cracks, not to be potential sources of weakness, should remain closed so that pavement slabs on both sides of a crack may act to sustain traffic loads near the crack. Decreasing temperature and other shrinkage continually act to widen a crack and distributed reinforcement is dimensioned to withstand this action by overcoming the subgrade resistance—"subgrade friction" on one side of the crack. The literature discloses that distributed reinforcement in adequate amount will prevent detrimental opening of transverse cracks as long as the steel is not stretched or broken. Edge reinforcement (marginal bars) is intended not so much to prevent opening of transverse cracks but rather to dowel the slab corners at the cracks. This load transfer function is generally intended with any concentration of reinforcing steel near pavement edges, such as "heavy-edge" welded wire fabric. Frequently, reinforcement has been concentrated near corners or transverse edges, critical parts for traffic loads.

Typical failures

Structural cracks in concrete highway pavements can be classified into some fairly distinct types. Fig. 3 shows a selection of typical

structural pavement cracks. The following notes are intended to give their characteristics in relation to reinforcement.

Exterior corner crack—(Fig. 3a) Corner cracks clearly can be linked with insufficient strength for traffic loads. Corner reinforcement "hair-pin bars" and doubled distributed reinforcement have both been used to reinforce exterior and interior pavement corners, but there are no experience records which permit an appraisal of the effectiveness of the reinforcement.

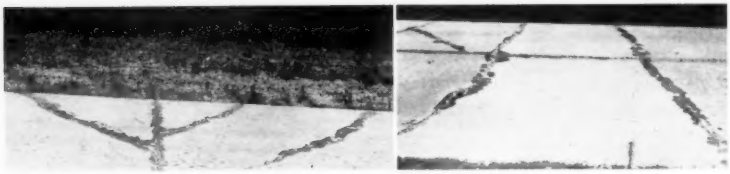
Blow-ups—(Fig. 3b) These are concrete buckling or shear failures occurring when longitudinal compressive stresses become excessive. In frequency they vary from none to several per mile of pavement. Their relationship to joints, reinforcement, and to infiltration of dirt into joints is still subject to investigation.[3] Some expansive aggregates have caused excessive blow-up.[4] Incorrect slanting joint installation can result in blow-up under relatively low compression. Reinforcement in amounts effective for crack control is known to have a beneficial influence because of the diminished soil infiltration in joints and cracks.

Interior corner crack—(Fig. 3c) Interior corners generally have not been strengthened by a thickened interior slab edge; corner cracking there is often related to structural weakness such as inadequate joint design. These cracks cannot be considered separate from the longitudinal and transverse joint behavior.

Transverse cracking—(Fig. 3d) This is the type of pavement defect which generally has been used for a quantitative determination of distributed steel reinforcement. As commonly found, transverse cracking has been assumed to be the result of tensile stress resulting from subgrade frictional resistance as the volume of the concrete tends to change with temperature or moisture change. Other factors may be of equal importance because transverse crack intervals have been found to vary from 10 ft to more than 100 ft in different pavements. Transverse cracks some few feet distant from a transverse joint appear to be a type of structural failure associated with "pumping" and subsequent loss of subgrade support along the joint. The extent to which longitudinal reinforcement actually prevents transverse cracking under any condition has not been conclusively demonstrated. There is general agreement among highway designers that distributed longitudinal reinforcement if present in sufficient amount postpones cracking and holds the cracks substantially closed; this action is referred to as *controlled cracking.*

Broken pavement—(Fig. 3e) This term refers to irregular cracks occurring in some localized area of a pavement, but not to general disintegration. It is common to find a statement in concrete pavement construc-

Fig. 3—Typical structural "failures" in concrete pavements. While none of these may be common, a clear understanding of their occurrence is essential to pavement design and function of reinforcement.

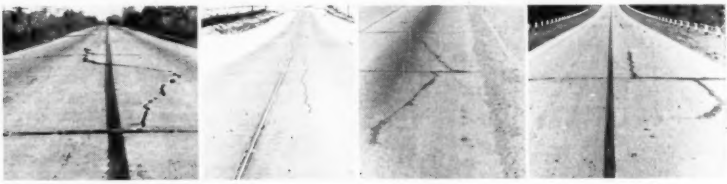


(a) (b)

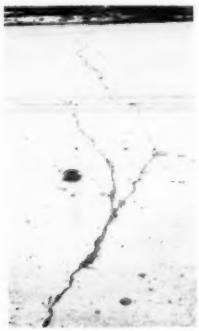
a. Exterior corner cracks at a transverse joint
b. Blow-up; the view shows the extent of patching because of the blow-up

(c) (d) (e)

c. Interior corner cracks at junction of transverse joint and longitudinal cleavage joint
d. Transverse crack caused by subgrade, volume change and temperature
e. Broken pavement caused by subgrade settlement over a culvert



(f) (g) (h) (i)



(j)

f. Longitudinal crack caused by excessive width (Differential vertical movement has later occurred under pavement beyond transverse joint)

g. Longitudinal crack caused by excessive restraint across longitudinal joint

h. Longitudinal crack at joint caused by restraint of concrete expansion near the edge of the pavement

i. Longitudinal crack at joint caused by insufficient edge strength or joint load transfer

j. Forked transverse crack caused by two $\frac{5}{8}$-in. deformed edge bars in bond with the concrete, the steel restraining concrete contraction

tion specifications such as "reinforcement used over culverts and soft subgrades." General pavement cracking can at times be related to the subgrade deficiencies. Localized changes in subgrade condition, such as transition from cut to fill, frequently appear to have a direct relation to erratic cracking. Whether or not reinforcement is effective in preventing structural failures for such subgrade conditions seems questionable; its value after crack occurrence is evident.

Longitudinal cracks because of excessive width—(Fig. 3f) An irregular longitudinal crack, generally attributed to pavement warping, was one of the most common structural defects of the early two-lane concrete pavements, built without a longitudinal center joint. The critical width for transverse warping at early age is not much over one lane. Generally, the crack appeared before the pavement was many days old, although on some sandy subgrades 18- and 20-ft wide pavements several years old have been seen with cracks extending only a few feet from transverse joints. The longitudinal cracks were controlled, and may have been prevented with reinforcement on some early pavements, but the crack generally is present on old 16-ft pavements. At least one state reports a substantial mileage of reinforced 14½ ft wide lanes of 9 in. uniform pavement recently constructed and thus far practically free from longitudinal cracking.

Longitudinal cracks because of excessive restraint across longitudinal joints—(Fig. 3g) The use of an excessive amount of tie bars across center joints has sometimes in the past restrained the joint and caused longitudinal cracks a short distance from the joint, commonly at the ends of tie bars. Recurrent designs for wide pavements show a tendency to increase tie steel across center joints to an extent where such cracking could occur. Longitudinal joints of weakened plane type of insufficient depth, or incorrectly installed, submerged below the surface, have sometimes caused this type of cracking. Pavement designs with large amounts of tie bars generally include also distributed transverse reinforcement.

Longitudinal cracks due to expansion restraint—(Fig. 3h) As a general characteristic these cracks show up 1 to 3 ft from the edge of the pavement and extend parallel to the edge, 5 ft or more from a transverse joint. They are believed to be more common in northern states. They have been linked with ice formation in the joint, with differential freezing of the subgrade under pavements partially cleared of snow, and with dirt filling the joint. Expansion joint fillers of insufficient length permitting a concrete plug to form at the edge of the pavement have been known to cause such cracks. The value of reinforcement for their prevention or control has not been analyzed.

Longitudinal cracks from transverse joint edges—(Fig. 3i) These are cracks extending some feet from a transverse joint and are far enough from the pavement edge to preclude infiltration as a cause. Such cracks have been recorded in a number of pavement surveys. Various reasons such as faulty joint construction, lack of or incorrectly aligned load-transfer devices, and excessive bending stress along the pavement edge have been advanced to explain their formation.

Forked transverse cracks—(Fig. 3j) Transverse cracks branching into Y- orientation toward a pavement edge (sometimes referred to as "crawfoot" cracking) occur as a characteristic behavior where relatively large reinforcing bars in bond with the concrete are used parallel to and near a pavement edge. They are believed to be the result of restraint to contraction of the concrete from a crack caused by large bars under high tension stress and bonded to the concrete.

REINFORCEMENT DESIGN AND TRANSVERSE JOINT PRACTICE

Highway Research Board report and related findings

In 1925 a report of an extensive study of the value of reinforcement in concrete pavements was presented for the Highway Research Board by C. A. Hogentogler.[5] In the course of his study, crack surveys covering a large number of concrete pavement projects throughout the United States were reviewed. Reinforcements in these pavements were studied in relation to *prevention* of cracking rather than with respect to *maintaining cracks closed.* This report has remained the most comprehensive publication on reinforcement for concrete highway pavements. It is unfortunate that except for a small percentage, the over 200 projects examined consisted of pavements built without longitudinal joints, and in many instances, with thickened center section. As a result, neither present pavement sections nor functions of reinforcement *after* cracking received specific consideration. The data were supplemented by information tending to show the effect of reinforcement as delaying but not decreasing the amount of cracking in a concrete pavement. This conception has remained unchallenged in the literature since that time.

Highway standards in review

Pre-war and post-war design practice of highway departments is summarized in Table 1, as obtained from data prepared by the Portland Cement Assn. The average weight of reinforcement shown in Table 1, is based on welded wire fabric, a number of states permit bar mats and expanded metal as well. Average reinforcement weight and joint spacings show trends only; Fig. 2 shows the weights of reinforcement and joint spacings for standard designs using wire fabric.

TABLE 1—REINFORCEMENT PRACTICES IN U. S. HIGHWAY DEPARTMENT STANDARD PAVEMENT DESIGNS

Year	Without steel	With edge or other heavy bars	With distributed reinforcement			
			Plans show, heavy traffic or special	Generally required	Quantity, ave.	
					Steel, lb per sq ft	Joint spacing, ft
	Number of highway departments					
1941	6	3	16	24	0.51	45
1947	8	2	18	20	0.58	49

RECENT DEVELOPMENTS

Observations of pavement reinforcing practices show wide variations between design standards, not traceable to traffic, climatic, material, and subgrade differences. During the last ten years, therefore, reinforcement for crack control has been receiving renewed interest. Naturally the two extremes of design are receiving major attention, on the one hand short slabs with no steel, on the other hand slabs of virtually unlimited length heavily reinforced.

Interest in pavement design is intensified by rapidly increasing truck axle loadings in excess of the loads for which the highway system generally was designed.[6] Another recent investigation concerns the use of reinforcement in airfield pavements for aircraft loadings of entirely unprecedented size.

Short highway pavements without steel

In these designs, transverse joints are spaced so close together that cracks between the joints are practically precluded. It is assumed that closely spaced contraction joints, without expansion joints, will generally remain so nearly closed that the slab edges remain interlocked and that surface water in injurious quantities is excluded from the subgrade. Unsatisfactory pavement behavior, because of water penetration through joints and cracks, pumping and loss of subgrade support is receiving widespread attention.[6] The soil variable is becoming increasingly more important as conditions of loading become more severe. Extended service behavior of short slabs with closely spaced contraction joints therefore deserves careful study for full appraisal of these designs.

Continuously reinforced highway pavements without joints

In these designs, extremely long slabs with continuous longitudinal steel near center depth are built without transverse joints. Experimental installations exist in Indiana, Illinois and New Jersey.[7,8,9]

Fig. 4—Continuously reinforced pavement on an experimental project in Illinois

The steel reinforcing is ¼ percent or more of the pavement cross section; a construction view is shown in Fig. 4. A 10-year performance report of the Indiana project, just released, contains the following data of general application to reinforced pavement design.[10]

In long pavement slabs the central portion remains constant in length, regardless of temperature. The ends of the slabs expand and contract with variations in temperature for some distance from the joints while subgrade resistance develops in the steel the stresses and strains necessary to offset the temperature length change. Restraint of daily expansion and contraction is not noticeable in slabs up to 75 ft long and apparently is complete for the center portion of slabs over 800 ft long.

The coefficient of subgrade resistance was much smaller than the value of 1.5 assumed in the design, and it was much smaller for the slow annual change than for the more rapid daily change in length. After 10 years service 70 percent of 154 short sections from 20 to 120 ft long were still uncracked. The report states: "If one were interested only in a minimum number of transverse cracks and joints, these data suggest that in reinforced concrete, transverse joints be spaced approximately 100 ft apart. However, as this investigation strikingly shows, a longer section with many transverse cracks can continue to be a strong durable structural unit after many years of heavy traffic service if it contains an adequate amount of longitudinal reinforcement."

Thick airfield pavements for heavy aircraft

Recently developed heavy aircraft impose wheel loads of 50,000 lb or more on airport pavements. Results of loading tests with a 150,000-lb single aircraft wheel load on plain and reinforced slabs from 12 to 25 in. thick have just been released.[11] Typical crack patterns for reinforced 12-in. slabs after 1000 loadings covering every spot on the slab are shown in Fig. 5. It illustrates the specific pattern of structural corner cracking in the 12-in. slabs and the absence of transverse cracking in these 50-ft slabs. Unreinforced 20-in. slabs withstood these

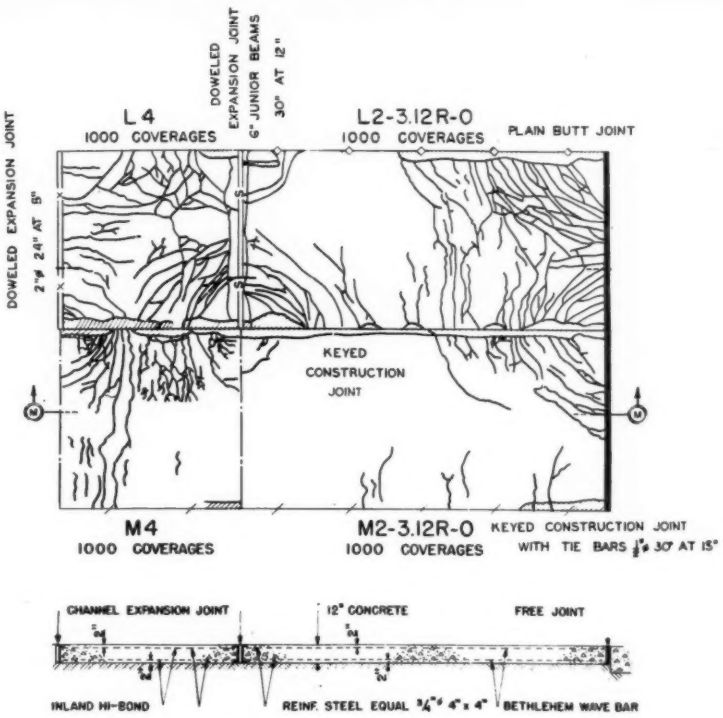


Fig. 5—Structural corner failures in 12-in. concrete slabs subjected to 150,000-lb aircraft wheel load at 1000 complete coverages of the wheel. Reinforcement sectional area in top and bottom was: slab L2-3, 0.41 percent each way, fabric; slab M2-3, 0.92 percent each way, bars

short-time loading tests without cracking. However, while complete failure quickly followed cracking of thinner unreinforced slabs, the reinforcement as shown multiplied the useful resistance of the cracked slabs to repeated loads. It is not here intended to compare reinforced and unreinforced slabs, but to show the need for careful attention to corner and edge stresses and the value of long slabs to limit corner stresses when designing reinforced slabs for heavy loads. By performance in this load test, extending over an 18-month period, 21-in. unreinforced slabs were judged a more satisfactory design economically than 14-in. reinforced slabs.

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PAVEMENT SURVEYS

Many unanswered questions remain before use and design of steel reinforcement in concrete pavements can be fully rationalized. The mass of information on performance of existing highway pavements in surveys or maintenance reports of highway departments is generally not available in published form. A few data from the literature are cited as examples which are of value in appraising the use of steel and advancing design criteria for reinforced pavements.

Pennsylvania data by Tebbs

These data[12] were published in 1931 and covered experience with various pavement sections and reinforcement practices from 1920 to 1930. The information does not permit detailed analysis but the conclusions are of interest. With transverse joint spacing about 180 ft, the average unbroken slab length varied from 48 ft (for thick-center fabric reinforced pavements over 7 years old on poor subgrade) to between 68 and 85 ft (for thickened-edge fabric reinforced pavements). A 61-ft contraction joint spacing was concluded to be possible for virtual elimination of transverse cracks. This joint spacing and 68 lb mesh reinforcement is still standard in Pennsylvania.

Rhode Island data by Spencer

These data,[13], published in 1946, cover transverse crack surveys by the Rhode Island Division of Roads and Bridges made annually since 1925. Most of the pavements had 42 lb mesh in 90 ft slabs, which at 15 years averaged somewhat less than one crack per slab. Significant are the results of crack surveys on one project with joint spacing from 40 to 90 ft, shown in Table 2. Of 106 50-ft slabs per mile, 18 cracked; of 59 90-ft slabs per mile, only 14 cracked. The present Rhode Island pavement standards include 46 lb mesh in 73½ ft slabs.

TABLE 2—CRACKING OF SINGLE-LANE 42-LB MESH REINFORCED PAVEMENT SLABS FROM 40- TO 90-FT LONG ON A RHODE ISLAND PROJECT

Joint spacing, ft	No. of slabs	Pavement length, ft	No. of transverse cracks	Average slab length, ft
40	16	640	None	40
50	280	14,000	47	43
60	33	2160	9	48
70	24	1680	7	54
80	4	320	4	40
90	132	11,880	31	73

Louisiana survey by Lehmann

A 900-mile pavement survey was made under the direction of H. L. Lehmann, Testing and Research Engineer, Louisiana Highway Department, during 1945. Mr. Lehmann has kindly made the complete report available to ACI Committee 325 for study. This comprehensive survey includes information on materials, cross section, joint and dowel details, curing methods and construction season, as well as age, traffic and core strength when surveyed. Slab failure counts are given separately for parts of a project on different predominating soil types; most of the typical slab failures are included separately. Table 3 gives the crack summary on all of these pavements for the different districts in Louisiana, divided into unreinforced, bar-reinforced, and fabric-reinforced pavements.

Almost all the pavements had 8-6-8-in. cross section and contained gravel aggregate or a mixture of gravel and crushed rock. Some plain pavements had no transverse joints, others had transverse joints 40 to 60 ft apart. The fabric-reinforced pavements had transverse joints 30 to 50 ft apart with 0.44 lb per sq ft steel, 75 percent of it longitudinal. The bar-reinforced pavement standard shows $\frac{3}{8}$-in. bar mat weighing 0.6 lb per sq ft, 69 percent longitudinal steel, for 30 ft spacing; bar-reinforced pavements with 40- to 60-ft joint spacing were surveyed as well, but exact reinforcement details are not available. The fabric-reinforced pavement design generally includes also dowels spaced 14 or 15 in. across the transverse joints. Approximately $\frac{2}{3}$ of the mileage is on A4 type soil and $\frac{1}{3}$ on A7 soil but aside from a somewhat greater occurrence of corner cracking on A7 soils, no significant difference in average behavior is noticeable for the two soils.

The condition of these pavements, unreinforced as well as reinforced, with but few exceptions is excellent as shown by Table 3. The average performance of the fabric-reinforced pavements was better in all respects than the plain pavements. The consistent decrease in all types of failures with light reinforcement indicates the need for evaluating reinforcement in relation to pavement failures other than transverse cracking. The only statistically discernible effect of pumping is the high ratio of half transverse cracks to whole cracks of 2:1 for some pumping pavements, although the reversed ratio is representative for pavements without indications of pumping.

In Fig. 6 transverse cracks and joints have been plotted in relation to joint spacing. Although the number of survey sections with 44 lb fabric-reinforced 50-ft slabs was only $\frac{1}{5}$ of comparable 30-ft slabs, they are sufficient to suggest that 50-ft reinforced slabs of the Louisiana design and for similar soil, climatic and traffic conditions may perform as well as 30-ft slabs, with a 40 percent saving in joints.

TABLE 3—SUMMARY OF LOUISIANA CRACK SURVEY

District	Length, miles	Age, years	Core test compres., psi	Ave. joint spacing, ft	Corner cracks per mile		Transverse cracks per mile	Longitudinal cracks		Blow outs per mile	Spalls per mile	Broken surface, sq ft per mile	Notes
					Ext.	Int.		No.	Length, ft, ave.				
Unreinforced pavements													(1)
1	64.4	14	7750	50	1.2	0.9	19.3	17.5	13	2.7	23.4	900	
2	120.8	14	7600	49	1.0	0.8	22.2	10.5	5	4.4	41.8	2230	
3	50.3	15	6040	—	0.5	0.2	74.3	4.4	13	4.1	4.2	1470	
4	49.3	14	6790	53	0.4	0.1	48.8	4.7	15	1.3	3.0	1020	(2)
5	115.0	13	7050	49	0.9	0.6	26.5	2.4	7	1.0	4.1	380	
6	90.9	14	6930	49	0.3	0	66.0	14.0	10	1.7	19.5	580	
7	5.7	14	8540	40	0	0	33.4	2.1	15	4.1	12.7	390	(3)
Ave.	496.4	14	7200	48	0.6	0.4	41.6	7.9	11	2.8	15.6	1000	
Bar-reinforced pavements													
1	11.3	14	7840	40	4.5	0.3	13.8	5.7	12	0.3	1.7	340	(4)
4	20.8	11	7090	40	0.1	0	20.1	6.4	10	0.6	4.0	80	
5	29.8	12	8060	52	2.0	0.08	59.0	0.9	10	2.5	5.5	3000	
6	11.6	12	8370	40	0.3	0	11.5	0.4	16	1.9	0.9	120	(4)
7	50.5	12	8130	39	0.5	0.1	12.5	0.5	8	0.6	3.2	1290	
Ave.	124.0	12	7900	42	1.5	0.3	22.4	2.8	11	1.2	3.1	970	
Fabric-reinforced pavements													
1	40.5	10	7250	36	0.1	0.2	5.4	1.9	5	1.4	18.5	10	
2	42.2	10	7700	30	1.0	0.2	15.4	2.6	3	0.4	10.9	135	
3	65.6	11	6900	34	0.2	0	6.8	3.0	13	0.6	3.7	10	
4	36.8	10	7100	36	0.1	0	4.9	0.1	4	0.5	3.4	725	(5)
5	37.1	10	7480	32	0.35	0.17	69.0	0	0	0.1	0.5	1460	(6)
6	12.7	9	7190	30	0	0	18.3	1.1	11	0	0.4	160	
7	43.9	9	7670	30	0.2	0	19.9	0.1	7	0.3	4.7	150	
Ave.	278.8 899.2	10	7300	33	0.3	0.1	20.0	1.3	7	0.5	6.0	380	

Notes: Transverse cracks and blow-outs of one lane only are included as one half.
(1) Most unreinforced pavements were listed as containing reinforcement over culverts and soft grade.
(2) Survey discontinued on one project after 6 miles because of slab overloading by heavy army traffic.
(3) One project part only. Numerous failure prevented completion.
(4) One project only.
(5) 22,140 sq ft of broken surface on one project, early settlement on high fill.
(6) Transverse cracks: 95 percent of half and 81 percent of whole cracks on one badly pumping project, 22 percent of mileage.

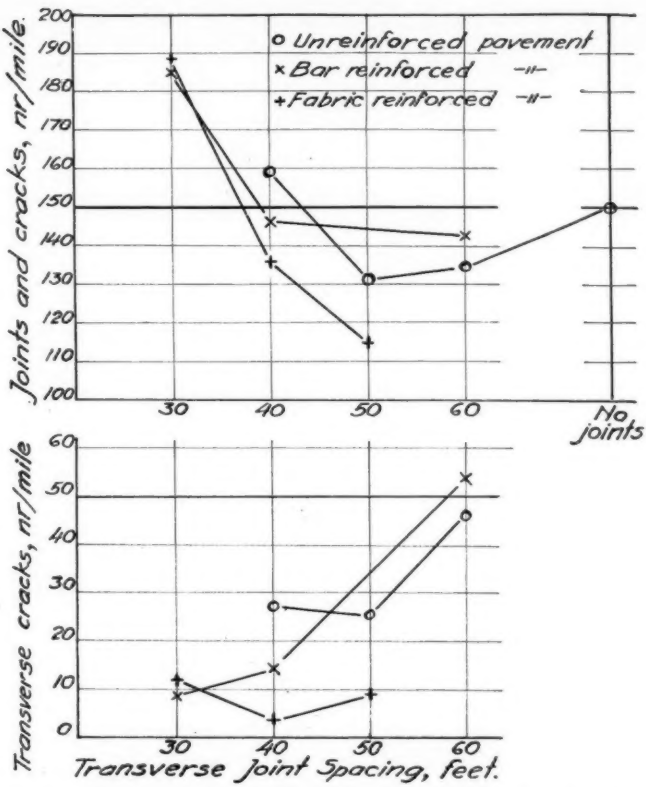

Fig. 6—Transverse cracks and joints in relation to transverse joint spacing for reinforced and unreinforced Louisiana concrete pavements from 10 to 14 years old, surveyed in 1945, average values

Many highway departments have made pavement surveys in recent years which offer valuable information on various aspects of pavement design; their publication in form permitting review is well justified. Considerable variations cannot be avoided but if the pavement stretches for survey are representative, the volume of data makes up for lack of uniformity between individually comparable sections.

CLOSURE

These observations on reinforcement practices in concrete pavements are intended to show in general terms the extent of our knowledge with the hope that they may stimulate continued research, study

(3) One project part only.
(4) One project only.
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and dissemination in published form of accumulated experience toward more complete understanding and correct economical use of this important design element.

ACKNOWLEDGMENT

This study has been prepared as a contribution to the work of ACI Committee 325, Structural Design of Concrete Pavements for Highways and Airports, Subcommittee 4, Design and Use of Steel Reinforcement. The paper is not a subcommittee official release. The author is indebted to H. L. Lehmann for making available the Louisiana Concrete Pavement Crack Survey.

Much valuable comment and advice in the preparation of the paper has been given by L. W. Teller, chairman of Committee 325 and by fellow members of Subcommittee 4: L. M. Arms, William Van Breemen, C. A. Willson, K. B. Woods and F. N. Wray.

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Do test cylinders cast in paper molds give a true picture of compressive strength?

Tests of Paper Molds for Concrete Cylinders*

By ROBERT A. BURMEISTER†

SYNOPSIS

Concrete test cylinders cast in a new type paper mold had a compressive strength lower than that predicted for the concrete mix used. Investigation showed that cracks and mechanical injuries to the outer shell of the concrete cylinder caused by movement of the paper stock during the first 24 hours of curing reduced the strength of the cylinder. To a lesser degree this was true also of the paraffined paper molds in common use for casting test cylinders, specimens cast in both types of paper molds showing lower strengths than test cylinders from the same mix cast in steel molds.

INTRODUCTION

The City of Milwaukee Testing Laboratory annually tests several thousand 6 x 12-in. concrete cylinders for compressive strength. The majority of these cylinders are made in the field for control tests of municipal paving projects, other governmental projects and for the local building industry. Paraffined paper molds have been used almost exclusively for this work because of their economy and convenience. Most of these paraffined paper molds were of the familiar type hereinafter designated as Type A, but late in the 1949 construction season a new type of paraffined paper mold was introduced. This latter mold, hereinafter designated as Type B, immediately gave trouble, for predicted concrete strengths were not attained and the concrete cylinders often exhibited small cracks which seemed to be caused by elongation of the paper mold during approximately the first 24 hours. This elongation of the mold was especially noticeable when the cylinders (in molds) were field cured in wet sand.

DESCRIPTION

Type A and B molds are cylindrical, 6 in. in diameter, 12 in. high and have walls made of what is generally described as paraffined paper

*Received by the Institute Jan. 9, 1950. Title No. 47-2 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 22, No. 1, Sept. 1950, *Proceedings* V. 47, Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Jan. 1, 1951. Address 18263 W. McNichols Rd., Detroit 19, Mich.

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or cardboard. Type A has a paper bottom and Type B mold has a metal bottom (Fig. 1). A paper technologist described the walls of the Type A mold as made of "bending chip board stock" and those of Type B as "jute container board stock." Tests on these materials are shown in Table 1.

TABLE 1—THICKNESS AND STRENGTH OF MOLD STOCK

	Thickness, in. (by a standard paper micrometer)	Bursting strength, psi (tested from inside of stock to outside by a Mullen Tester)
Type A	0.085	320
Type B	0.078	535

Tests on the mold stock for paraffin content, moisture and absorption are given in Table 2. Elongation of molds when filled to 1 in. from top with water is shown in Fig. 2.

TABLE 2—PARAFFIN CONTENT AND MOISTURE ABSORPTION OF MOLD STOCK

	Paraffin content, percent by wt. of mold stock*	Moisture absorption, percent by wt. of mold stock†	Moisture absorption, lb per mold†
Type A	23.7	40.7	0.25
Type B	19.3	53.6	0.30

*Average of 3 samples, 2 x 2 in.
†Average of 6 samples, entire mold used, except that metal bottom in Type B was corrected for. Moisture amount based on molds "as received," no oven drying.

Mold A showed no apparent leakage when water filled, provided the joint between the bottom cap and walls was sealed in the laboratory with paraffin. Mold B was watertight as received. Of course, in both types some water was lost by evaporation through the walls.

CONCRETE TESTS

A total of 130 concrete cylinders were made out of two batches of concrete. These batches of concrete were made as nearly alike as possible, except that Batch 1 was made with cement No. 1 and Batch 2 was made with cement No. 3. Concrete was mixed in a commercial transit mixer and delivered to the laboratory. Mix data on this concrete is given in Table 3.

Cylinders were made and cured in groups of ten for Type A, Type B and steel molds according to the following schedules:

Fig. 1—Paper molds for concrete specimens, Type A (left), Type B (right)

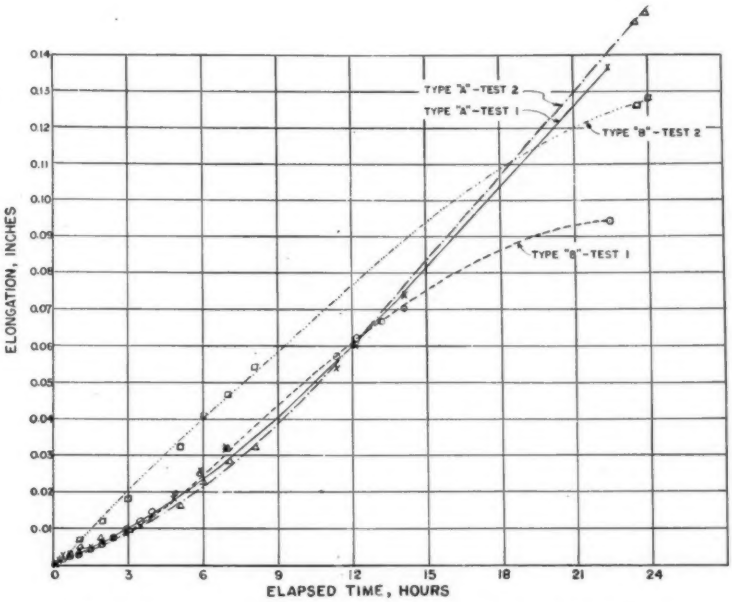

Fig. 2—Elongation of paper molds when filled to 1 in. from top with water

Batch 1

Cylinders made* and cured in moist room.

Cylinders made in laboratory mixing room, stored there for 24 hours and then cured in moist room, Type A and B only.

*Moist room water spray units shut off during this operation.

Batch 2

Cylinders made* and cured in moist room.

Cylinders made in laboratory mixing room, stored there for 24 hours and then cured in moist room, Type A and B only.

Cylinders made while molds were immersed to 1 in. from top in water bath, stored there for 24 hours, then cured in moist room, Type B only.

Cylinders made in laboratory mixing room, stored there for 24 hours and then cured in moist room, Type A and B only (reparaffined).

All molds were removed after 24 hours. At this time it was noted that for steel molds, tops of mold and concrete were practically flush; but that for paper molds, the paper walls protruded above the concrete and there was noticeable adhesion between paper and concrete. Storage for the remaining 27 days was in the moist room. Compressive strength tests at 28 days are reported in Table 4. Cylinders were capped with Hydrocal. For computing the compressive strengths, the nominal end area of 6 in. diameter cylinder (28.3 sq in.) was used, no corrections were made for minor variations in diameters of the cylinders. Spot checks for average diameter of steel molds were 5.96 in., for Type A mold 5.97 in. and for Type B mold 5.96 in.

DISCUSSION

Although early field examinations indicated that the Type B mold was the sole producer of cracked cylinders, these tests show that cracked cylinders may be gotten with Type A also. Fig. 3 (cylinder No. 44-105)

*Moist room water spray units shut off during this operation.

TABLE 3—CONCRETE DATA

	Batch 1		Batch 2	
Cement	Type I, Brand No. 1		Type I, Brand No. 3	
Sand	Source "J"		Source "J"	
Gravel No. 1 (¼-¾ in.)	Source "H"		Source "H"	
No. 2 (¾-1½ in.)	Source "H"		Source "H"	
Batch wts. (lb per cu yd)	Wet	Dry	Wet	Dry
Cement	517	517	517	517
Sand	1222	1192	1227	1189
Gravel No. 1	1120	1101	1130	1095
No. 2	1100	1081	1130	1095
Water—added at mixer	209		196	
—Total*	223		236	
Cement content (sacks per cu yd)	5.49		5.43	
Air content, percent by vol. (pressure method)	1.2		1.8	
Slump, in.	3		3½	
Workability	Fair +		Good −	

*Includes free water on aggregates.

shows a field cylinder made with Type B mold; note the deep crack. Cylinder No. 38, Batch 1, (Fig. 4) was made with a Type A mold and the type of small crack it exhibits has previously escaped notice. Fig. 5 shows a cylinder made in a steel mold; note the smooth texture.

From Table 4 it is apparent that concrete compression test cylinders made in Type A or Type B cardboard molds show lower strengths than do those made in steel molds.

The most plausible theory that may be advanced for the lower strengths of cylinders made with paper molds is:

Elongation of the cardboard during the early curing period causes cracking and mechanical injuries to the surface or shell of the cylinder of concrete. The circumferential increase in dimension of the mold is apparently not enough to free the mold from the concrete. The mold elongates as the concrete sets or hardens, causing surface movement of the concrete until such a time when the concrete becomes strong enough to resist. Simultaneously, of course, the paper becomes weaker as it nears the moisture saturation point.

In support of this theory it will be observed that the greatest reductions in strength occurred when Type B molds were used. The paper in these molds was almost double in strength (Table 1) of that in the Type A molds and thus had a greater capacity for doing damage. Also note that the elongation of both types of molds when water filled (see Fig. 2) was approximately 0.01 in. after 3 hours and approximately 0.03 in. at 6 hours. This elongation is enough to account for the cracks in the cylinders which cannot be accounted for in any other way.

Corollarially, it is apparent that reductions in strengths were considerably affected by the amount of curing water present. Using the average strengths for cylinders made in steel molds for each batch as standard, reductions in strength for the paper molds under different curing conditions are as follows:

Type B molds, Batch 1—when made and cured in moist room for first 24 hours lost 660 psi or 13.6 percent; when stored in laboratory mixing room for first 24 hours lost 510 psi or 10.5 percent.

Type B molds, Batch 2—when stored in water for first 24 hours lost 1010 psi or 21.2 percent; when made and cured in moist room for first 24 hours lost 530 psi or 11.1 percent; when stored in laboratory mixing room for first 24 hours lost 560 psi or 11.7 percent.

Type A molds, Batch 1—when made and cured in moist room for first 24 hours lost 400 psi or 8.2 percent; when stored in laboratory mixing room for first 24 hours lost 360 psi or 7.4 percent.

Type A molds, Batch 2—when made and cured in moist room for first 24 hours lost 150 psi or 3.1 percent; when stored in laboratory mixing room for first 24 hours lost 490 psi or 10.3 percent.

Considering curing specifications for cylinders for the first 24 hours as prescribed in A.S.T.M. C31 which are conformed to by the curing

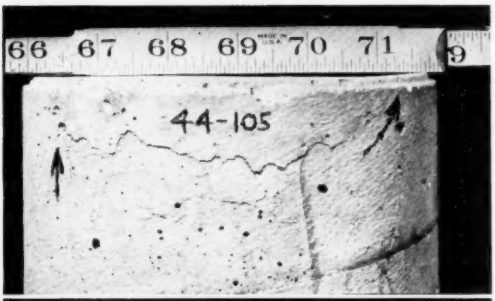

Fig. 3 — Field cylinder made in Type B mold. Photographed upside down as made. Note crack marked by arrows. This crack extends obliquely into cylinder end for about ¼ of its length (on right side as photographed)

Fig. 4—Crack in cylinder made in Type A mold (No. 38, Batch 1)

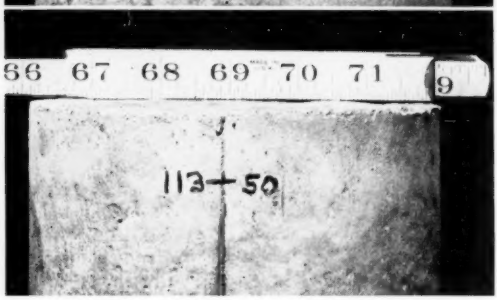

Fig. 5—Cylinder made in steel mold (No. 50, Batch 1). Note smooth texture. Vertical line was made by mold joint

conditions for those cylinders which were stored in laboratory mixing room for first 24 hours,

For mold Type B, Batch 1, loss in strength was 10.5 percent
For mold Type B, Batch 2, loss in strength was 11.7 percent

Ave. 11.1 percent

For mold Type A, Batch 1, loss in strength was 7.4 percent
For mold Type A, Batch 2, loss in strength was 10.3 percent

Ave. 8.8 percent

TABLE 4—COMPRESSIVE STRENGTH FOR 6x12 IN. CYLINDERS AT 28 DAYS

Batch No. 1

Made and cured in moist room									Made and stored 24 hr. in mixing room, then stored in moist room					
Type A mold			Type B mold			Steel mold			Type A mold			Type B mold		
Cyl. No.	Strength psi	% avg.	Cyl. No.	Strength psi	% avg.	Cyl. No.	Strength psi	% avg.	Cyl. No.	Strength psi	% avg.	Cyl. No.	Strength psi	% avg.
21	4240	95	1	4150	99	41	4650	96	31	4310	96	11	4380	101
22	4410	99	2	4270	102	42	4380	90	32	4130	92	12	4140	95
23	4170	94	3	4330	103	43	4890	101	33	4630	103	13	4530	104
24	4570	103	4	3830	89	44	5030	105	34	4490	100	14	3980	92
25	4750	107	5	4240	101	45	4800	99	35	5040	112	15	4520	104
26	4940	111	6	4390	104	46	5020	103	36	4410	98	16	4220	97
27	4710	106	7	4580	109	47	4760	98	37	4470	99	17	4070	94
28	4650	104	8	3740	89	48	4970	102	38	4690	104	18	4830	111
29	3720	83	9	4480	107	49	5080	105	39	4400	98	19	4810	111
30	4390	99	10	4020	96	50	4940	102	40	4460	99	20	4010	92
Avg	4460			4200			4860			4500			4350	
Maximum deviation from average, percent														
Below	17			11			10			8			8	
Above	11			9			5			12			11	
Batch No. 2														
51	4690	102	11	4280	100	71	4970	104	41	4280	100	1	4450	106
52	4130	89	12	4210	99	72	4660	98	42	4310	101	2	4180	99
53	4590	99	13	4000	94	73	4840	101	43	3890	91	3	3740	89
54	4730	102	14	4220	99	74	4500	94	44	4050	95	4	4480	106
55	4190	91	15	4670	110	75	4740	99	45	3910	91	5	4450	106
56	4650	101	16	3950	93	76	5040	106	46	4950	116	6	4280	102
57	4760	103	17	4050	95	77	4770	100	47	4070	95	7	4160	99
58	4880	106	18	4750	112	78	4830	101	48	4720	110	8	4350	103
59	4670	101	19	4170	98	79	4420	93	49	4150	97	9	4100	97
60	4880	106	20	4120	97	80	4900	103	50	4450	104	10	3890	92
Avg	4620			4240			4770			4280			4210	
Maximum deviation from average, percent														
Below	11			7			7			9			11	
Above	6			12			6			16			6	

Batch No. 2

Molds in water first 24 hr. then moist room			Made and stored 24 hr. in mixing room, then stored in moist room					
Type B mold			Type A mold reparaffined			Type B mold reparaffined		
Cyl. No.	Strength psi	% avg.	Cyl. No.	Strength psi	% avg.	Cyl. No.	Strength psi	% avg.
21	3330	89	61	4460	107	31	4240	100
22	3670	98	62	4210	101	32	4380	104
23	3820	102	63	3940	94	33	3790	90
24	3210	85	64	4100	98	34	4400	104
25	4350	116	65	4000	96	35	4550	108
26	3820	102	66	4130	99	36	4360	103
27	*		67	4180	100	37	3900	92
28	3810	101	68	4380	105	38	4310	102
29	4110	109	69	4100	98	39	4530	107
30	3710	99	70	4260	102	40	3750	89
Avg.	3760			4180			4220	
Maximum deviation from average, percent								
Below	15			6			11	
Above	16			7			8	

*Specimen not tested, saved for exhibit.

Considering curing conditions analogous to those in the field when cylinders are buried in wet sand for the first 24 hours, *i.e.*, cylinders made and cured in moist room,

For mold Type B, Batch 1, loss in strength was 13.6 percent
For mold Type B, Batch 2, loss in strength was 11.1 percent

Ave. 12.3 percent

For mold Type A, Batch 1, loss in strength was 8.2 percent
For mold Type A, Batch 2, loss in strength was 3.1 percent

Ave. 5.6 percent

Considering the extreme in curing conditions, that is for the cylinders made and cured in water bath for first 24 hours, for mold Type B, Batch 2, loss in strength was 21.2 percent.

The steel molds gave the most consistent results for individual cylinders; in Batch 1 nine of the ten cylinders tested had compression strengths deviating no more than 5 percent of the average, the remaining cylinder had a strength deviating 10 percent from the average; while for Batch 2 none of the 10 cylinders deviated more than 7 percent of the average.

Reparaffining of paper molds of both types had little effect on strengths; evidently the paper was still highly absorptive and subject to growth when wetted.

While these tests may place paper molds for concrete cylinders in an unfavorable light, a redeeming factor is that the errors obtained in using them are always on the lower or conservative side of the true compressive strength. It is apparent, however, that there is a definite need for a disposable mold for field control use; one that is cheap, easily removed, nonabsorptive, and constant in form during the first 24 hours.

CONCLUSIONS

1. Concrete compression test cylinders made in paper molds show lower strengths than do those made in steel molds.

2. The reduction in strength is apparently due to cracks and mechanical injuries to the outer shell of concrete of the cylinder caused by movement of paper stock during the first 24 hours of curing.

3. Type B molds cause greater reductions in strength than Type A.

As an aid to the preparation of a formal report, ACI Committee 212 invites extensive discussion of the papers presented here.

Admixtures in Concrete*

SYNOPSIS

Five papers by members of ACI Committee 212, Admixtures, are, because of their common general subject, presented together.

Various admixtures are discussed briefly as an introduction to more detailed treatment of air-entraining materials. The relative merits of admixtures and interground agents are considered. Optimum ranges of air content for different structural uses are given with particular reference to pavements.

The advantages and disadvantages of several types of admixtures used in the fabrication of various concrete products, such as building block, cast stone, pipe, cribbing and curbing, are discussed. The admixtures considered are classified into the following groups: accelerators, air-entraining agents, gas-forming agents, water repellent agents, and workability agents.

The factors affecting bleeding characteristics and workability of fresh concrete are reviewed and the effect of admixtures on these properties is assessed.

Present knowledge of admixtures in counteracting alkali-aggregate reactions is reviewed. It is emphasized that further studies may revise thinking in this field.

The effectiveness of various concrete admixtures in inhibiting the capillary flow of water and the flow of water under pressure is considered. The types of admixtures included in the discussion are accelerators, soaps, butyl stearate, finely subdivided dry materials, mineral oil, workability agents, and a miscellaneous group of proprietary compounds.

Introduction

By W. T. MORAN†

The mission of ACI Committee 212 is to study, assemble and report to the Institute information on the effect of various admixtures, including air-entraining agents, on the properties of concrete. With this objective

*Presented at the ACI 46th annual convention, Chicago, Ill., Feb. 21, 1950. Title No. 47-3 is a part of the copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 22, No. 1, Sept. 1950, *Proceedings*, V. 47. Separate prints are available at 60 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Jan. 1, 1951. Address 18263 W. McNichols Rd., Detroit 19, Mich.

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in mind, committee members are now actively engaged in reviewing available material with possible future publication of a report in mind.

This report, which incidentally is an expansion or continuation of the committee's 1944 report, is presented as a single source compilation of information in as simple terms as possible. If successful in this alone it will be an accomplishment when it is recognized that the use of admixtures does not simplify in any way the technology of portland cement concrete and that the subject itself is a complex one. Recognizing the ever-increasing importance of admixtures, particularly as related to pozzolanic materials and air-entraining agents, the committee is moving cautiously before presenting to the Institute recommendations as to any particular usage.

The committee is now actually engaged in reporting on the advantages and limitations of admixtures (1) in grouting operations, (2) to counteract alkali-aggregate reaction, (3) in correcting grading deficiencies, (4) use in concrete products, (5) in underwater construction, (6) to overcome "false" set, (7) to increase resistance to freezing and thawing, (8) to obtain high early strength and (9) as integral waterproofing materials. The present purpose is to present informal previews of five of the nine subjects just described. Each of these five subjects will be presented by various members of the committee but it is pointed out, however, that the individual presentations which follow have not of necessity the final benediction of our committee membership, and for this reason each author is essentially on his own. In this way, it is hoped that some instructive information will be made available, while at the same time the general membership will be given an opportunity to discuss some of our ideas. The committee would like to gather as many different criticisms, suggestions and ideas as possible in order to incorporate them, if possible, in the final report to the Institute.

The Use of Admixtures to Increase Resistance to Freezing and Thawing

By F. H. JACKSON*

Prior to the discovery about 10 years ago that the resistance of concrete pavements to surface scaling could be materially improved by incorporating a small quantity of air into the mixture, the use of admixtures to improve durability was not, in general, considered good

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practice. Most engineers believed that, although the use of such materials might prove advantageous under certain conditions (as, for example, to accelerate the hardening process or to dampproof the concrete), the value of using admixtures specifically to improve durability was distinctly questionable. It was the general consensus that more positive results would be secured by adhering strictly to the fundamental principles of making good concrete with the basic materials (cement, aggregates and water) than by the use of admixtures.

It is true that there have appeared from time to time numerous proprietary materials which, according to their sponsors, definitely improve the durability of concrete. Some of these materials contain a water-reducing agent which tends to reduce the quantity of water required for a given consistency, thereby improving both strength and durability. Admixtures of this general type were available long before the beneficial effects of air entrainment were recognized. Some of them may have entrained small amounts of air. However, they appear to have been used very little in concrete for the specific purpose of promoting durability.

In addition to those water-reducing agents, many very finely divided mineral admixtures have a beneficial effect upon the workability of a harsh concrete and thereby may improve durability indirectly by facilitating the handling and placing of the concrete, and in this way increasing its uniformity. Materials of this general type have also been promoted for a long time. However, as in the case of admixtures of the water-reducing type, they have never been generally accepted. In some cases, where the original harshness or lack of workability is due to the peculiar nature of the aggregate, it may be found that the use of this type of admixture is justified. However, if the harshness is merely due to some deficiency in aggregate gradation or to faulty mix design, the best solution usually is to correct the trouble by changing the grading and/or redesigning the mix, rather than by attempting to mask the deficiency by the use of an admixture. Admixtures are never justified merely to correct avoidable deficiencies in good engineering practice.

The effect of pozzolanic admixtures on the various properties of concrete, including durability, is being studied by a number of agencies. There is evidence that the use of such materials, properly controlled, and in combination with air entrainment, will definitely improve durability and at the same time make possible substantial reductions in cement content.

EFFECTS OF AIR ENTRAINMENT

During the past 10 years the attitude of the engineering profession towards admixtures in general and admixtures for improving durability

in particular has undergone a marked change. This has been due almost entirely to the discovery of the benefits of air entrainment. It was formerly believed that the best protection against damage by frost action was to use a dense, impermeable mix well placed and thoroughly cured. These basic principles still hold. However, it is now known that, even when all of these rules are followed, resistance to frost action can be still further improved by incorporating into the mix a small amount of air (usually from 3 to 6 percent by volume of the concrete) in the form of minute bubbles well distributed throughout the mass. Furthermore, it is becoming increasingly evident that this additional protection is needed, especially in cases where freezing conditions are severe or where (as in the case of pavements) common salt or calcium chloride is used for ice removal.

Improved durability

Just why the resistance of air-entraining concrete to weathering should be so much better than even the best ordinary concrete is still a question. Technicians, while agreeing as to the fact, differ as to the reasons. However, one rather generally accepted explanation is that these minute air bubbles act as so many tiny cushions within the hardened mass, thereby absorbing the pressure which would otherwise be exerted by the expanding ice. In any event, experience in both laboratory and field during the last 10 years has demonstrated the superior durability of air-entraining concrete and there should be no hesitancy in using it under conditions where severe natural weathering is a problem.

Air entrainment materially alters the properties of both the plastic and the hardened concrete. Air-entraining concrete is considerably more plastic and workable than ordinary concrete. It can be handled and placed with less segregation and there is less tendency for bleeding and water gain. Obviously these properties indirectly aid in promoting durability by increasing uniformity, eliminating planes of weakness at the top of vertical lifts, etc.

Decreased strength

Air entrainment, while improving both workability and durability, has an adverse effect upon strength. Within the range of practical air contents, the decrease in strength is about proportional to the amount of air entrained. However, for most types of exposed concrete, the slight reduction in strength is more than balanced by the improved resistance to frost action. Moreover, the reduction in strength will rarely exceed 15 percent in the case of compressive strength and 10 percent in the case of flexural strength. These figures are for equal cement content but with the sand and water content of the air-entrain-

ing concrete reduced to the extent permitted by the increased workability of this type of mix. In cases where structural design is based on compressive strength, it is considered that the high factor of safety ordinarily used in design will insure adequate strength, particularly when we consider the improvement in uniformity and durability. In pavements where we do not have so high a factor of safety, the reduction in flexural strength may be taken care of, when necessary, either by increasing the thickness of the slab or by increasing the cement content. However, no instances have so far come to the writer's attention where the reduction in strength due to air entrainment has seriously affected the structural stability of a concrete pavement.

AIR-ENTRAINING ADMIXTURES

There are now on the market some 30 or more commercial admixtures for which air-entraining properties are claimed and some of these are being extensively used. There are many materials, including the natural wood resins, fats, various sulfonated compounds, oils, etc., which have the property of entraining air in concrete. Some of these materials, as for example the resins, are insoluble in water and must be saponified before they can be used as admixtures. Soluble salts of the resins are now available commercially so that it is no longer necessary to saponify them in the field.

Before specifying or permitting the use of any air-entraining admixture in concrete, the user should satisfy himself (1) that the material actually does function as an air-entraining agent and (2) that none of the essential properties of the concrete (strength, volume change, etc.), are seriously impaired. The American Society for Testing Materials has developed a tentative method of testing air-entraining admixtures for concrete (A.S.T.M. Designation C 233-49T) which is an excellent guide as to the nature and extent of the information which should be available before a particular admixture is approved.

AIR-ENTRAINING CEMENT

Air-entraining concrete can also be made by using an air-entraining portland cement. The American Society for Testing Materials has issued a tentative standard specification (C 175-48T) for this type of cement. Air-entraining portland cement is similar to ordinary portland cement except that it contains an air-entraining agent which has been interground with the cement during its manufacture. Engineering opinion is divided as to the relative merits of admixtures versus air-entraining cements. Each method has its advantages and disadvantages. There are certain practical advantages in using an air-

entraining cement. The necessity of handling an additional material at the mixer is avoided. On the other hand, it is possible to exercise close control of the quantity of air more readily through the use of a material added at the mixer. This is because the amount of entrained air in any given case depends upon a number of variables in addition to the cement. These variables include richness of mix, consistency, character and grading of aggregates, type of mixer, mixing time, weather conditions, etc. When an admixture is used it is possible for the engineer to take all these factors into consideration and to adjust the amount of admixture on the job to secure the specified air content. This can be done readily by means of trial mixes, using an air-meter for determining directly the air content of the plastic mix.

It is not as easy to control the air when using an air-entraining cement. The amount of air-entraining material ground into the cement is fixed, so far as the engineer is concerned, and cannot be changed by him. Nevertheless, it is frequently possible to make satisfactory adjustments. For example, if too much air is being entrained, the difficulty may often be corrected by reducing the proportion of sand in the mix or the amount of water, or both, and still maintain the required workability. Considerable adjustments in this direction is possible due to the improved workability imparted by air entrainment and full advantage should be taken of this fact. It is not so simple to increase the air content. However, in cases where the air is too low and adjustments in mix are not feasible, it is always possible to bring the air content up to requirements by using an admixture. This practice is followed frequently by state highway departments and other agencies which desire to provide for the use of air-entraining cements in general but which recognize that the occasional use of admixtures may be necessary.

CHOICE OF METHOD

It will be seen from the above that the choice between the two recognized methods of entraining air depends almost wholly on whether the engineer wishes to exercise the precise control which is possible when admixtures are used or whether he would rather make the necessary mix adjustments and other changes that are frequently required when using air-entraining cements. Both methods are being used extensively and both are giving satisfactory results. However, regardless of which method is employed, the important thing is to be sure that the concrete contains the correct amount of air. This can be done either by direct measurement using an airmeter or by the so-called gravimetric method described in A.S.T.M. Method C 185-47T. Determinations should not only be made prior to construction but also periodically during con-

struction to insure that the correct air content is being maintained at all times.

OPTIMUM AIR CONTENT

The optimum air content varies with the type of concrete and the maximum size of the aggregate. Engineers are not fully agreed as to the exact quantitative relations but the following broad classification may be considered as essentially correct.

Classification	Optimum air content, percent
Mass concrete. Maximum size of coarse aggregate 3 to 6 in. Cement content 3 to 4 sacks per cu yd	3.0 ± 1.0
Pavement concrete. Maximum size of coarse aggregate 1½ to 3 in. Cement content 5 to 6½ sacks per cu yd	4.5 ± 1.5
Structural concrete. Maximum size of coarse aggregate ¾ to 1½ in. Cement content 5 to 7 sacks per cu yd	5.0 ± 2.0

The above comments apply primarily to concrete structures subject to alternate freezing and thawing but not to frequent or heavy applications or salt for ice removal. In the case of pavements, it was formerly the practice to provide skid resistance on icy roads by mixing the calcium or sodium chloride with sand or cinders prior to application. Wherever this practice is still followed, experience has indicated that concrete with 3 to 6 percent entrained air will give full protection. However, it is now becoming common practice in some states to apply the raw salt directly to the pavement to completely melt the ice and there is evidence to indicate that even air-entraining concrete will not withstand this treatment indefinitely. At the moment, engineering opinion differs on this point. Some highway engineers, particularly those in New York State, feel that additional protection such as a surface seal of some sort is necessary where heavy applications of the raw salts are employed. Others claim that the primary reason for the deterioration which has been observed is the fact that too little air has been entrained. This school is definitely of the opinion that much better results would be secured if the air content were held closer to the maximum allowable value of 6 percent than is now common practice. The fear of certain highway engineers that strength or drying shrinkage may be seriously affected if the air content is allowed to go too high has undoubtedly operated to keep the average value rather low in actual practice.

Use of Admixtures in Concrete Products

By BRUCE E. FOSTER*

From the scarcity of published material, it would appear that admixtures have been employed only to a limited extent in the manufacture of concrete products. However, admixtures of a number of types have found application, and definite advantages resulting from such application have been reported.

The optimum conditions for the most efficient use of any admixture can usually be obtained only after considerable experiment with a particular application. The addition may frequently require modification in the proportions of the other constituents and unless many combinations are explored full advantage of its beneficial properties will probably not be realized. Following the outline established in the committee's 1944 report,† the various general types of admixtures are listed in alphabetical order, and the results which may be expected from each group are discussed.

ACCELERATORS

Materials which accelerate the hardening and promote early strength development of concrete prove advantageous in the manufacture of a variety of concrete products. The early attainment of strength in a building block, for example, reduces the curing period, compensates in part for slow hardening during cold weather, and decreases the time required to produce a fully mature block. Similar advantages may be obtained in the manufacture of other concrete products. An accelerator may be used either with or without high-temperature curing. During warm weather accelerators should be used judiciously so as not to produce too rapid a set. Many plants employ only the high-temperature curing during summer months, and a combination of high-temperature curing with an integral accelerator during winter months.

The most commonly used accelerator, calcium chloride, is generally used in amounts up to 2 percent but never more than 3 percent, by weight of cement. For best control it should be dissolved in water to make a standard-strength solution which may be measured into the batch. Some instances of efflorescence have been attributed to calcium chloride in the mix.

AIR-ENTRAINING AGENTS

Air-entraining concrete has come into wide use in recent years because of its increased resistance to the action of freezing and thawing.

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†Report of ACI Committee 212, "Admixtures for Concrete," ACI JOURNAL, Nov. 1944, *Proc.* V. 41, pp. 73-86.

In the manufacture of concrete products, however, the increase in durability obtained with air entrainment is generally of secondary importance as compared with other beneficial properties.

Quantitative data in the literature are lacking, but the improved workability resulting from entrained air has been credited with several advantages in the manufacture of concrete block. With any of the usual manufacturing methods, the use of air-entraining concrete permits greater compaction, resulting in denser blocks. Appearance of the blocks is improved. Edge tear is reduced, and the blocks strip cleanly with sharp edges and corners. Of equal importance is the improvement in surface texture which may be obtained especially where aggregates deficient in fines have been employed. Blocks made with entrained air also have less absorption and present greater resistance to water passage.

Of prime concern to the block manufacturer is a reduction in breakage of green blocks. In block manufacture a dry mix must be employed since the product is removed from the mold immediately after compaction. The addition of an air-entraining agent permits the use of more water, thus giving a water-cement ratio closer to the optimum value for maximum strength with the given proportions of dry materials. The fresh blocks have a "rubbery" consistency which results in fewer cracks and less breakage during handling at early ages.

Similarly, satisfactory results are obtained with other concrete products. In cast stone the plasticizing effect of entrained air reduces bleeding and segregation and makes possible more exact reproduction of mold contours.

The use of air-entraining concrete in the fabrication of pipe results in better flow of concrete around reinforcing, less tendency to displace reinforcing during compacting processes, easier stripping of molds without distortion of the pipe, sharper edges, better appearance, and less permeability.

In some installations of concrete products such as cribbing and curbing, there is considerable exposure to freezing and thawing action. As already pointed out in this group of papers the use of adequately prepared and controlled air-entraining mixes is the best guarantee of resistance to this destructive action.

The application of lightweight aggregates to concrete products is increasingly rapidly. Lightweight aggregate mixes are generally harsh, and hence the use of air entrainment with them is particularly advantageous. In many cases the reduction in bleeding and cement segregation resulting from air entrainment is great.

In general beneficial results may be obtained by the discreet addition of entrained air in both lean and rich mixes with any grading or type of aggregate, including those with light weight. The greatest improvement is obtained, however, in harsh mixes deficient in fines.

The simplest method of obtaining entrained air is by the use of an air-entraining cement. Such a cement might, where necessary, be blended with a non-air-entraining cement to get closer control in the final mix. For best results it is probably desirable to add an air-entraining admixture at the concrete mixer, because different amounts are necessary to produce the optimum results in various products and methods of making the same product. Relatively large amounts of air-entraining admixtures are needed for best results in making block while small amounts are preferred in cast stone manufacture. The Packer-Head Process of pipe manufacture benefits from relatively large amounts of air entrainment, the dry-tamp process from small amounts. The proper amount in any particular case must be determined by experiment. Care should be exercised to avoid over entrainment since the strength of the product may be greatly reduced by an excessive air content.

GAS-FORMING AGENTS

The application of gas-forming agents for the production of lightweight building blocks is gaining some attention. The usual agent is aluminum powder, which when added to the mortar reacts with the hydroxides produced by hydration of the cement to generate small bubbles of hydrogen throughout the mass. In some cases an alkali such as caustic soda also is added in small amounts to accelerate the gas formation. The rate of formation is also greatly affected by temperature. It will be seen, then, that careful control and adjustment of the added materials must be made if a uniform product is to be obtained. Under proper conditions of fabrication the product has light weight and good heat insulating properties. Low strength makes it unsuitable for load bearing partitions. The blocks should not be placed in contact with the ground because of their low resistance to water penetration.

WATER-REPELLING AGENTS

Much of the water which penetrates walls built of block units passes through the joints. Little benefit therefore can accrue from an attempted waterproofing of the blocks themselves. Since water repellent agents are not effective in reducing the flow of water under pressure, their use is not indicated in such products as concrete pipe. However, such additives, particularly butyl stearate, have merit when employed for the reduction of efflorescence and of staining in cast stone.

WORKABILITY AGENTS

Any agent which will increase the workability of the mix may prove advantageous. Greater compaction, better flow around reinforcement, finer reproduction of mold detail including sharp corners and edges, and decrease in permeability, may be expected. Also a decrease in breakage of green products is probable. Workability agents fall in three classes.

Finely divided minerals

Harsh mixes deficient in fines may be improved by the addition of finely divided materials. In the manufacture of concrete block, fly ash has given good results. Since fly ash has pozzolanic properties it may be used to replace part of the cement as well as to increase the fines in the mix. A 20 percent replacement of cement by fly ash has been reported to give satisfactory results. The addition of an excessive amount of fine material will considerably increase the required amount of mixing water. Such a condition should be avoided as it results in lowered strength and increased shrinkage due to drying.

Organic plasticisers

Certain organic compounds or mixtures, marketed under proprietary names, improve the workability by chemical means. The sulfonated type generally increases air entrainment and part of their effectiveness can be attributed to this property. The carbohydrate type increases workability without noticeable increase in air entrainment. Reports on the effectiveness of both types are inconclusive. Both have been reported to make possible a 20 percent reduction in cement content without loss of strength; however, there are other data showing no advantages in their use.

Aerating materials

The important effect of entrained air on workability has been discussed above under air entraining agents.

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The Use of Admixtures for the Correction of Aggregate Gradation

By T. C. POWERS*

Concrete engineers have used admixtures to improve the workability and bleeding characteristics of fresh concrete and sometimes to modify the properties of the hardened concrete. Most of the materials so used are mineral powders as fine as or finer than portland cement. They therefore serve to "correct" the properties of the paste. Since the distinction between the fine material in the paste and that in the aggregate is more or less arbitrary, mineral admixtures may also be considered as correctives for aggregate gradation.

Some mineral powders are relatively inert chemically; others are pozzolanic or cementitious. Their chemical characteristics are of secondary importance so far as the effect on workability and bleeding characteristics of fresh concrete is concerned.

Recently, air-entraining admixtures have been introduced. Their influence on plasticity and bleeding characteristics is similar to that of mineral powders.

BLEEDING AND PLASTICITY

Among the various factors determining the bleeding characteristics and the degree of plasticity of fresh concrete, the amount of surface area of the solids per unit of water volume is the most important.[1,2] When the ratio of surface area to water is too low, the paste has a thin, watery consistency. Consequently, at a given slump the volume of paste in the concrete only slightly exceeds the volume of voids in the aggregate. This means that the aggregate particles are only slightly separated by paste layers and hence that the mix has a low capacity for plastic deformation[2,3] and a high tendency to segregate.[4] It denotes also a high incidence of particle interference according to Weymouth's criterion.[5]

When the ratio of surface area to water is too low, the rate of bleeding is relatively high. Moreover, most of the bleeding does not appear at the surface; that is, the aggregate particles settle for a short period and then establish point-to-point contacts that prevent further settlement. The watery paste continues to bleed within the pockets defined by the aggregate particles, leaving layers of water at the undersides of the particles. Thus, with mixtures having the characteristics just discussed, bleeding tends to reduce the homogeneity of the concrete.

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UNDESIRABLE EFFECTS OF BLEEDING

In extreme cases the lack of homogeneity is manifested by open fissures under the aggregate particles large enough to be visible in a cross section of the concrete. As early as 1917 Edwards[6] published photographs of cross sections showing the separation of the paste from the undersides of the aggregate particles. In 1927 (and at various later times) Gilkey[7] discussed this phenomenon. In 1929 Powers[8] published experimental evidence of its adverse effect on strength and watertightness. Collier[9] in 1930 referred to this phenomenon in explaining deviations in strength at constant water-cement ratio. Ruettgers, Vidal, and Wing[10] (1935) showed that at a given water-cement ratio and stage of curing the concrete was more permeable the larger the maximum size of the aggregate. These results seemed to demonstrate that the fissures under the aggregate particles contribute in a major way to the permeability of the concrete. Casey[11] (1937) published photomicrographs showing fissures under the aggregate particles for concretes made with cements having different bleeding characteristics. In 1939 Powers[1a] published experiments showing that the loss in strength at a given water-cement ratio was a function of the degree to which bleeding is hindered by particle contacts.

PARTICLE SEPARATION

The undesirable effects of bleeding described in the foregoing paragraphs can be ameliorated by increasing the ratio of surface area to water in the paste. This generally increases the stiffness of the paste and at a given slump effects a wider separation of the aggregate particles in the concrete. The ratio of surface area to water volume may be increased by increasing the amount of cement (lowering the water-cement ratio of the paste) or by supplementing the cement with a suitable admixture.

However, the degree of particle separation required to eliminate undesirable bleeding effects is high. It requires the volume of the mix to exceed that of the dry rodded aggregate by as much as 50 percent. To effect such a degree of separation requires a paste of very low water-cement ratio or the use of an excessive amount of admixture. It is necessary therefore to base the selection of materials and proportions on a compromise; some potential strength and impermeability must be sacrificed to obtain lower water contents and consequently better volume-change characteristics.[12] It seems best, all things considered, to design concrete mixes so as to obtain the least possible water content for the desired consistency.

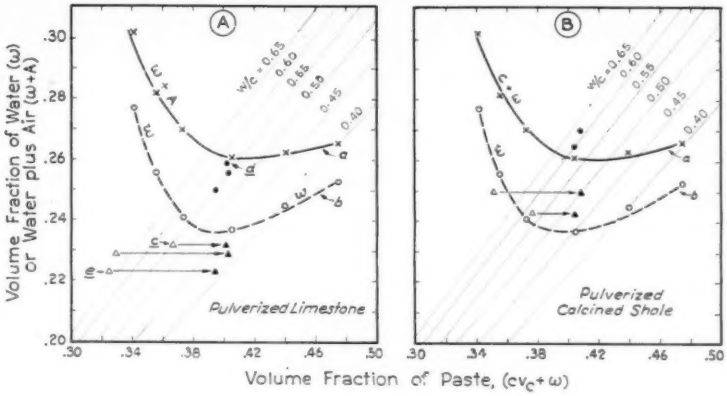
Although under some circumstances the use of admixtures lowers the amount of voids in the concrete, admixtures should not be regarded as

void-fillers—at least not in the sense of providing small particles to fill the voids between the larger particles. A mixture of either cement and water or cement, admixture, and water forms a soft, plastic paste—technically a weak solid[3]—which envelops the aggregate particles and, in the course of the mixing process, separates them. Thus, in a sense, the function of the admixture is to *increase* the voids in the aggregate and in so doing to increase the capacity of the mix for plastic deformation. Adding 1 percent of a material of high specific surface such as bentonite together with the necessary amount of water will effect the same increase in aggregate-particle separation as 10 or 15 percent of a mineral powder of ordinary specific surface. This shows that the volume of paste produced by the admixture rather than the manner in which the particles fit into the voids of the aggregate is the factor controlling the effectiveness of the admixture.

In 1931, G. M. Williams[4] reported: "The relative efficiency of admixtures in increasing workability is measured by the relative volumes of paste produced when equal weights of admixtures are made into pastes of approximately equal flowability." Practically the same conclusion was published in 1932 by Powers,[13] who reported data indicating that the workability of concrete was determined by the quantity and consistency of the paste rather than by the characteristics of the powder or powders of which the paste is made. This study indicated also that for any given materials there is an optimum paste consistency at which the water requirement of the concrete for a given mobility is at a minimum. In still later studies McMillan and Powers[14] in reporting a study of admixtures stated: "Any of the powdered materials tested, or blends of two materials, when used in such proportions as to produce a paste of given consistency, gave, when combined with the same gradation of aggregate, concrete mixes of substantially equal workability." A method was given for computing the required amount of admixture on the basis of the water contents of admixture-pastes tested separately. Such computations serve as a first approximation to be checked by trial batches. John C. Sprague[15] also found that the relative amount of admixture required to control the bleeding capacity of concrete was roughly proportional to the relative amount of water required by the admixture for "normal" consistency.

SELECTING AND PROPORTIONING ADMIXTURES

Fig. 1 illustrates the relationships pertinent to the problem of selecting and proportioning mineral admixtures. The relationships illustrated apply to a particular cement and aggregate. A diagram of the same kind may be developed for any given combination of materials.



C = Weight of cement. x—x - Total voids-no admixture (ω+A)
v_c = Sp. vol. of cement. o—o - Water content - no admixture
W = Volume of water. • - Voids vs. cement paste-with admixture
A = Volume of air. △ - Water vs. cement paste- " "
▲ - Water vs. total paste - " "

Elgin sand, fineness modulus 2.90
Remolding Effort, 43 jigs

Fig. 1—Typical interrelationships of water, air, water-cement ratio and paste content for mixes with and without admixtures

Curve *a* is the relationship between total voids (water plus air) and the paste content, paste being defined here as the sum of the volumes of water and cement. Curve *b* is the relationship between the water content and the paste content. The vertical distance between these curves at any paste content is the air content of a particular mix. The variation in paste content was produced by changing the cement content and adjusting the water content so as to maintain a fixed consistency as indicated by the remolding test.* The water-cement ratio of any given cement-water paste may be estimated from the superimposed reference lines.

For this particular case the water requirement reaches a minimum at W/C = 0.45 by weight, with the paste content at 40 percent of the volume of the mix. This amount of paste is to be compared with 32 percent, the voids in the dry rodded aggregate.

It is evident that for these materials the cement paste having a water-cement ratio of about 0.45 has the optimum consistency. When thinner

*Similar relationships may be established on the basis of the slump test or on equal consistency as judged by an experienced operator.

pastes were used, the quantity of paste was smaller, but the water and void content of the mix was greater than the possible minimum.

If the voids in the aggregate are reduced by increasing the maximum size, the optimum paste content is likewise reduced. However, as found empirically by Abrams[16] and analytically by Charles T. Kennedy,[17] for a given maximum size the optimum grading is coarser than that which gives minimum voids in the dry aggregate; the greater the desired particle separation, the coarser should be the gradation. The optimum water-cement ratio of the paste (optimum, that is, with respect to water content of the mix) will also differ with different aggregates and cements. Therefore, Fig. 1 is not to be taken for more than an example of a specific case.

If it is desired to produce concrete having a higher water-cement ratio than that required for optimum paste consistency, that can be accomplished without increasing the water content of the mix by supplementing the cement with a suitable admixture. For example, in Fig. 1A point *c* represents a mix containing 0.367 parts (by absolute volume) of cement paste having a water-cement ratio of 0.55 by weight. To this paste was added enough powdered limestone to increase the paste content to 40 percent as indicated by the solid triangle to which the point *c* is tied. Notice that for a plain mix of the same water-cement ratio as that represented by point *c* the water content of the concrete would be 0.239 as compared with 0.232 for the mix containing an admixture, a difference of 1.4 gal., or 12 lb of water per cu yd. The other open triangles in Fig. 1A show the results of adding pulverized limestone to mixes containing smaller amounts of cement than the mix represented by point *c*. Here again the water content of the mix containing the admixture is substantially below that of the corresponding plain mix. The water content of the mix represented by point *e*, for example, was 0.025 less than that of the plain mix of the same water-cement ratio, a difference of 5 gal., or 42 lb of water per cu yd.

These particular combinations of cement and admixture produced pastes having the necessary consistency with a lower water content than the paste made with cement alone. This result is not generally to be expected, however. With the admixture represented in Fig. 1B, the water content was increased by using pulverized calcined shale as an admixture. In this case, the water content of the concrete containing 40 percent of cement-admixture paste was higher than that of concrete containing the same amount of cement paste. The greater the ratio of admixture to cement in the paste, the higher the water content.

In general the higher the proportion of solids in a paste of given consistency the lower the water content of the concrete at a given degree

of plasticity. If a mixture of cement and admixture contains the same volume of solids as cement paste of the same consistency, the use of the admixture (in correct proportion) will have little or no influence on the water requirement of the concrete as compared with the water requirement of the best mix without admixture. If the solid content of the cement-admixture paste is lower than that of the cement paste having the same consistency, the use of the admixture is liable to result in an increase in the water requirement of the mix, and vice versa. It follows, therefore, that admixtures giving pastes of relatively high solid content are to be preferred.

Particle shape and specific surface are the principal factors controlling the water requirement of an admixture and hence the relative solid content of the paste. For materials composed of particles of similar shape the higher the specific surface the lower the solid content of a paste having a given consistency. (Exceptions may be found among materials that do not differ greatly in specific surface.*) It is therefore advantageous to avoid admixtures having a high specific surface, such as bentonite or diatomaceous earth. Admixtures of low specific surface are to be avoided also. They are ineffective because they do not increase the ratio of surface area to water-volume sufficiently for satisfactory plasticity and bleeding characteristics.[1b]

With either of the admixtures represented in Fig. 1, the water-cement ratio of the paste is lower when the correct amount of admixture is used than it is when the admixture is omitted. For a given admixture the correct amount depends on the difference between the desired water-cement ratio and the water-cement ratio of the cement paste having optimum consistency. If the desired ratio is greater than that of the optimum cement paste, the quantity of admixture required is greater the greater the difference. Fig. 1 shows that if the desired water-cement ratio is equal to or less than that of the cement paste having optimum consistency, the use of admixtures is liable to increase the water content of the mix. If cement alone is added to lower the water-cement ratio to the desired value, the water content of the mix will be increased according to curve *b* (for the particular case represented). Adding an admixture would increase the water content of the mix according to the water requirement of the admixture. Unless its water requirement is zero or negative, the admixture will surely increase the water requirement of the mix. In general, admixtures are advantageous only when used in mixes that would contain less than the optimum volume of paste without the admixture. This consideration should

*Although not yet fully established by experiment, the factors that control paste consistency appear to be the same as those that determine bleeding characteristics of the paste. These factors are: specific surface, particle shape, forces of interparticle attraction, and chemical reactivity with water.

restrict their use to concretes containing pastes of relatively high water-cement ratio.

CONCLUSION

These general statements are a reliable guide for choosing among materials that have widely different characteristics. However, among similar materials the choice should be made on the basis of their respective effects on the water requirements for a given volume of paste in the concrete as determined from trial mixes. More experimental work needs to be done to establish the optimum characteristics of mineral admixtures. The optimum probably depends upon the desired water-cement ratio of the paste.

Entrained air stiffens the cement paste and, in effect, increases its volume.[18,19] Therefore, some of the foregoing discussion applies to entrained air as well as to mineral powders. Entrained air lowers the water requirement, the amount of lowering being relatively great for mixes containing pastes of relatively high water-cement ratio.[20]

If the aggregate is so deficient in small particles that the optimum gradation for the desired paste consistency cannot be produced, the volume and consistency of the paste can be increased by the use of admixtures to a point suitable for the best grading that can be produced. The committee does not recommend this procedure if it can be avoided. The use of finely divided admixtures to correct deficiencies in fine sand sizes is liable to result in concrete of unnecessarily high water content.

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Use of Admixtures to Counteract Alkali-Aggregate Reaction

By W. T. MORAN

The use of admixtures for the specific purpose of counteracting alkali-aggregate reaction is of rather recent development, and for this reason this presentation should be considered only as a review of the most recent knowledge on the subject. The general matter in the discussion which follows will obviously be subject to future changes as improved techniques and further studies are made.

The alkali-aggregate reaction, which is attributed to the interaction of the alkalies in portland cement and certain siliceous types of aggregates, has resulted in excessive expansion and surface cracking of the resulting concrete. The term alkalies as applied to portland cement refers to the sodium and potassium compounds present in relatively small proportions and expressed as the oxides. When this particular type of expansion phenomena was first observed by Stanton in 1938, the only apparent remedy or palliative was the use of portland cement of low-alkali content (0.60 percent or less computed as Na_2O).

To date field experience and service records of concrete in which cements of low alkali contents and reactive aggregates were employed have not indicated that excessive expansions will occur. However, a considerable number of long-time laboratory tests have indicated that destructive expansions are possible with some aggregates when used with comparatively low-alkali cement. It has also been ascertained that certain natural or artificial pozzolans are capable of reducing the expansions caused by high-alkali cements when used in combination with reactive siliceous aggregates. Although not an application of admixtures, it should be mentioned that the use of limestone as coarse aggregate in "sweetening" the mix has been reported as overcoming this reaction. It should be noted that those materials found to be effective in counteracting the alkali-aggregate reaction are pozzolanic materials. However, the converse is not necessarily true, some proven pozzolans showing little ability to prevent expansion. For this reason, it is necessary to have available some relatively rapid means of evaluating the efficiency of a given material for the purpose intended. It should also be pointed out that in this discussion only the effect of certain pozzolans on reducing the alkali-aggregate reaction is considered. It should not be forgotten that other beneficial properties may be obtained in concrete through the judicious selection of a suitable pozzolan.

EVALUATION TEST

At the present time, the most reliable known suitable method of evaluation of alkali reactivity of pozzolans is a reduction-in-expansion test, which is a direct method for measuring the ability of the material to reduce expansion caused by this reaction. The basic requirement for a test of this nature is a uniform supply of reactive aggregate. Synthetic glasses have been found to be highly reactive; a source of readily available material of uniform composition being Pyrex No. 774. This is the standard pyrex glass used for laboratory and domestic ware and for industrial piping. It is available as "lump cullet" from the Corning Glass Works, Corning, N. Y. The artificial aggregate is prepared by crushing, screening and washing a fine aggregate composed of 20 percent of each of the five sizes from 8 mesh to 100 mesh. Fines passing the 100 mesh screen are discarded because they tend to reduce the expansion. Those portions passing the 100-mesh sieve are so highly reactive that they react with the alkali in the cement in the same fashion as an effective pozzolan. Consequently this reduces the amount of available alkali remaining in the set mortar to cause a reaction. In fact, this characteristic is true of most reactive aggregates if they are ground fine. It might also be added that when pozzolans are used to control this reaction, care must be taken to grind sufficiently and to use them in sufficient proportion

to assure that an opposite effect will not be obtained, although this condition might only occur in an extreme case.

To determine the effectiveness of any particular pozzolan, it is used to replace 25 percent by volume of the cement. The mortar bars (1 x 1 x 11¼ in., with an effective gage length of 10 in.) are fabricated of high-alkali cement and glass aggregate. The bars are stored under sealed moist storage and the expansion determined at 14 days. Acceptable materials usually show a reduction of about 75 percent in the amount of expansion obtained as compared with specimens made without them.

The reduction-in-expansion test, although showing considerable promise, is subject to some further considerations. Such questions as how adequately does this test indicate delayed excessive expansions, and what is the effect of temperature variations during the test period must still be answered. The question of supply of a standard high-alkali test cement will be difficult to resolve bearing in mind the effect of cement fineness and difference in proportions of sodium or potassium for any given cement. A possible solution to this problem might be to limit the alkali content of the test cement to 1.0 percent and make up the deficiency by the addition of sodium hydroxide solution. Another approach could be the intergrinding of the dry oxides of sodium and potassium with portland cement or fusing these oxides with cement, quenching and grinding to obtain a controlled high alkali content.

TYPES OF POZZOLANS

Pozzolans which have been found to reduce the expansion caused by alkali-aggregate reaction in concrete or mortar may be classified in three categories: (1) materials containing amorphous siliceous and aluminous substances which include some opals and highly opaline rocks, certain volcanic glasses, diatomaceous earth, and calcined clays of the Kaolin type. Also included in this category are fly ashes, some of which significantly reduce the expansion caused by alkali-aggregate reaction and others which do not; (2) calcined clays of the montmorillonite type. These are crystalline and contain the calcium ion, for example bentonite clays; and (3) combinations of the above two categories which include such materials as siliceous shales, *e.g.*, Monterey shale and altered pumicites which are mixtures of montmorillonite clays and volcanic glasses.

The amount of suitable pozzolan required in a concrete to control this reaction will vary with individual aggregates and alkali content of the cement; however, it has been found that ample protection can normally be obtained by replacement quantities ranging from 20 to

35 percent by weight of the cement. These quantities are usually compatible with other physical requirements of the concrete such as strength, workability, etc., and are usually in an amount sufficient to permit economies in their use. The foregoing should, however, be considered as a generalization since it has also been ascertained that certain materials when finely divided and of high opal content (*e.g.*, certain diatomaceous earths and opaline cherts) will prevent expansions by using cement replacements less than 15 percent. Consideration must also be given in the use of these latter materials to the usually attendant increased water requirement.

OTHER ADMIXTURES

With respect to the effect of other types of admixtures on the alkali-aggregate reaction, there is some evidence that air-entraining admixtures reduce the expansion and that calcium chloride increases it, although these effects are but slightly pronounced. There is also some evidence that aluminum powder reduces this expansion as the result of gas entrainment.

CONCLUSION

In connection with the use of an admixture to counteract alkali-aggregate reaction, the question can logically arise whether an effective material is sufficient to counteract this reaction *per se*. That is, could a high-alkali cement be used with a highly effective admixture? Also, the question can be asked, can a less efficient admixture, from the point of view of the alkali-aggregate reaction, be used with a low-alkali cement? Until these points can be resolved, it is probably the most logical practice in the light of present knowledge to use only the most efficient or proven materials in conjunction with a low-alkali cement where reactive aggregates are of necessity to be employed in the work.

Use of Admixtures as Integral Waterproofing and Dampproofing Materials

By BRUCE E. FOSTER

Water passes through concrete by two different processes. In the one, water under pressure and in contact with one surface of the concrete is forced through either relatively large or small channels connecting the inner and outer surfaces. Permeability measurements under sizeable pressures give an indication of the ability of a concrete

to resist this type of flow. The reduction of permeability by use of an admixture or other process is termed in this report waterproofing. In the second type, the passage of moisture through concrete may take place in the absence of any appreciable external water pressure simply by the action of capillary forces which draw the water into and throughout the mass. Evaporation from surfaces exposed to unsaturated air, and the constant replenishment of moisture from the surface in contact with water, result in a flow of moisture through the concrete. The reduction of moisture transfer of this second type is termed dampproofing. Measurements of absorption and capillary rise as well as of permeability under low pressures give an indication of the dampproof qualities of a concrete.

Some admixtures may have merit for both waterproofing and dampproofing. Other agents which act by virtue of being water repellent may reduce the capillary forces, but owing perhaps to frothing action during mixing, increase the permeability to water under pressure.

Many investigators are of the opinion that concrete, carefully designed, placed and cured, will be satisfactorily waterproof without the use of any admixture; and that leakage due to cracks, joints and honeycombing is apt to be of much greater magnitude and concern than leakage through the sound mortar. However, since the usual design, placing and curing leave much to be desired, the use of a well-chosen waterproofing admixture may prove advantageous where low permeability is required, especially for concrete made from a lean mix.

Even though the best concrete practices are adhered to and the resulting concrete is impervious to water under pressure, passage of moisture due to capillary action may take place if the concrete is in contact with moisture or damp earth. In cases where even slight passage of water is objectionable the addition of a dampproofing agent may be advisable.

The use of such admixtures should in no case be considered as a substitute for careful choice of materials and mixes, for skilled workmanship, or for adequate curing. Rather it should be looked upon as an insurance which may to a greater or less extent correct deficiencies which otherwise might occur due to deviation from good concreting practices. The value of such insurance in any particular case will depend on the actual deficiency in the concrete, the character of its exposure to water or dampness, and the admixture selected for incorporation in the concrete. Obviously in no case can an admixture be expected to compensate for cracks or large voids in the concrete, although it may minimize the probability of their occurrence.

Since the effectiveness of an admixture in a given application depends upon many factors, including not only the placing and curing of the

concrete, but also the properties of all the individual materials and their proportions, the choice of an admixture in a particular instance should be made only after experimental work has been conducted.

Many preparations are sold for use as integral waterproofers or dampproofers. They vary widely in composition and in the nature of their action. Specific preparations are not discussed in this report, but the various general types of integral materials are considered. They are discussed from the standpoint of composition of the additive, of effect on strength and other physical properties of concrete, and of probable effectiveness as waterproofing or dampproofing agents.

TYPE 1—CALCIUM CHLORIDE SOLUTIONS

This section applies to calcium chloride solutions only. It does not apply to the many commercial preparations of calcium chloride containing small quantities of other materials, which will be considered later.

The listing of calcium chloride as a dampproofing agent has been questioned. However, it is included as such because a number of proprietary materials sold as waterproofings consist of little more than a calcium chloride solution.

Calcium chloride in amounts up to 2 percent by weight of cement may be used safely in concrete except at elevated temperatures or with cements of high alkali content. Its most important function is that of an accelerator, serving to increase the rate of cement hydration and of strength development. Little difference is found in the permeability or absorption of similar concretes with and without calcium chloride, except that there is some evidence showing that the use of 3 to 5 percent of the salt decreases the permeability at early ages.

Under certain conditions the use of calcium chloride may prove beneficial. The importance of adequate curing in the making of impervious concrete is well recognized. The use of an accelerator shortens the period during which curing must be continued for best results. This fact should not be construed as a recommendation that the employment of an accelerator makes good curing unnecessary, but rather that the effects of bad curing practices will be lessened.

TYPE 2—SOAPS

Admixtures classified as "soaps" are salts of fatty acids, usually calcium or ammonium stearate or oleate. In commercial preparations the soap content is usually 20 percent or less, the balance of the solid material being lime or calcium chloride. The quantity of soap incorporated in concrete should not exceed 0.2 percent by weight of the cement. Larger percentages result in excessive frothing action with

consequent increase in void space and decrease in strength. From the standpoint of dampproofing of concrete, a soap functions primarily as a water repellent. Its relative effectiveness in this respect is dependent upon its uniformity of distribution throughout the concrete.

Tests have shown that, in general, concretes into which soaps have been incorporated are more permeable to water under pressure than similar ones without such treatment. This has been attributed to the foaming action noted during mixing, but the explanation is not wholly consistent with permeability results obtained with air-entraining concrete.

It should be mentioned that water repellents frequently improve the workability of concrete, contributing to greater ease of placement. Their use may thus be beneficial in that they lessen the probability of the formation of large voids or pockets.

Under conditions of moderate exposure to moisture, judicious use of water-repellent integral materials is beneficial. Penetration of water from exposure to rains, or through capillary action in the case of concrete in contact with soil, may be reduced greatly.

The use of calcium chloride, in conjunction with water repellents, tends to compensate for the reduction in concrete strength at early ages. Ultimate strengths, however, are normally lower than for plain concrete. A word of caution with respect to curing procedures with water-repellent concrete should be given. Intermittent curing cannot be used because once the concrete has dried out it cannot readily be wetted again.

TYPE 3—BUTYL STEARATE

The action of butyl stearate in concrete is similar to that of the soaps in that it produces a water repellent effect. Unlike the soaps, it does not have a frothing action with the concomitant decrease in density of the concrete. It may be used, therefore, in much greater quantity than the soaps. To attain greatest uniformity of distribution throughout the concrete it is usually added as an emulsion. A quantity of emulsion found suitable is that which gives the equivalent of 1 percent of butyl stearate by weight of the cement.

Butyl stearate is superior to the soaps as a water repellent in concrete. It is noticeably more effective against penetration of water by capillary action when under moderate exposure, and it does not increase the permeability, as the soaps frequently do. When used in recommended amounts its effect on the strength of concrete is negligible.

TYPE 4—FINELY SUBDIVIDED DRY MATERIALS

Materials in this category may either be inert or may react with other constituents of the mix. Reactive materials are usually pozzolanic, combining with lime liberated during the hydration of the cement. Examples of pozzolanic materials are diatomaceous earth, fly ash, certain calcined shales, pumice, and some finely ground slags.

The degree of waterproofing achieved by the use of finely subdivided dry materials is closely related to the properties of the particular concrete mix. If the cement content of the cement-water paste is relatively low or if the aggregate is deficient in fines, the addition of the finely subdivided materials will probably be highly beneficial, decreasing permeability and increasing strength. However, if the concrete mix is rich in cement and the aggregate properly graded, added fine materials will probably be detrimental, since the increased surface area increases the water demand, resulting in a less dense and a weaker concrete. The above comments apply whether the added materials are inert or reactive. Regardless of quality of mix, reactive materials enjoy an advantage over the inert materials in that some additional strength results from the reaction.

An increase in cement content is more effective in decreasing the permeability of concrete than is the addition of any other fine material. Within limits, its use would be recommended in preference to other fine materials, but the cement content should not become excessive. In massive types of installation the desirability of maintaining heat evolution and subsequent shrinkage at a minimum requires that the cement content be kept low. In such cases, therefore, finely subdivided materials, preferably pozzolanic in character, may be advantageously used. Also, in any structure, an excess of fine material including cement may lead to objectionable properties of the concrete and should not be resorted to as a substitute for proper grading of the aggregates.

Tests have shown that finely divided materials have little merit as dampproofers.

TYPE 5—OIL

Heavy mineral oil has proved effective both in rendering concrete water repellent and in reducing its permeability. Oil so used should be selected with care. It must be a fluid petroleum product containing no fatty or vegetable oils which, through saponification, might adversely affect concrete strength; it should contain no petroleum residuals which emulsify with alkali and impair strength; it should be of such a viscosity (SAE 60) as to be readily miscible with concrete or mortar while possessing sufficient body to render the concrete dampproof.

Oil additions equaling 5 percent by weight of cement are only slightly detrimental to concrete strength and have proved effective against water under pressure.

TYPE 6—WORKABILITY AGENTS

Since the waterproofing qualities of concrete depend to a great extent on successful uniform placing of the material, an agent which improves the plasticity of a given mix without causing deleterious effects, or which limits bleeding and thereby reduces the number of large voids might properly be classified as a waterproofing admixture.

Such admixtures are well covered in the 1944 report of this committee, but may be briefly reviewed here. In addition to finely divided minerals which have already been discussed there are two general types.

Water-reducing agents

Certain organic compounds or mixtures have been marketed as water-reducing agents under proprietary names. In addition to increasing workability, the sulfonated type generally entrains some air and decreases the bleeding, while carbohydrate salts in combination with other active or inert materials affect the workability favorably but do not cause air entrainment or reduce bleeding.

Aerating agents

Air-entraining concretes have greater plasticity and bleed less than those without entrained air. The reduction in number of large voids left by poor compaction or caused by collection of bleeding water under aggregate results in less permeability. The view has been expressed that in addition to acting as a workability agent the small disconnected voids offer a barrier to the passage of water. Air-entraining concretes have lower absorption and capillarity than those without air and hence air entrainment may be classed as both an effective waterproofing and dampproofing agent.

TYPE 7—MISCELLANEOUS MATERIALS

Some proprietary powders, pastes and liquids available on the market as integral waterproofers cannot be classified under types thus far discussed. These products are made up of various combinations of materials such as (1) barium sulfate, calcium and magnesium silicates and a fatty acid; (2) finely ground silica and napthalene; (3) colloidal silica and a fluosilicate; (4) petroleum jelly and lime; (5) cellulose materials and wax in an ammoniacal copper solution; (6) silica, lime and alum; (7) coal tar cut with benzene; and (8) sodium silicate together with an organic nitrogenous material.

The chemical or physical actions of these miscellaneous materials on concrete are varied. Their effect on the strength of a properly designed concrete is usually detrimental. The waterproofing effectiveness of those containing water repellents, finely subdivided materials or oils, can be estimated from the foregoing discussions of such materials. Based on available test data, none of the miscellaneous materials enumerated above are appreciably effective in dampproofing concrete.

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Discussion of a paper by F. H. Jackson:

The Use of Admixtures to Increase Resistance to Freezing and Thawing*

By M. SPINDEL†

Both the Introduction by the Chairman of Committee 212 and the five papers by its members signify the important part which admixtures in concrete are already playing and are expected to play even more beneficially in the future. It is important to make quite clear that admixtures are of similar importance to mortar and concrete as are cement, aggregate, and water. Therefore, admixtures must be dealt with on the same sound basis of theoretical and practical knowledge.

This, of course, does not mean that the complex mechanism of chemical and physical reactions of all inorganic and organic substances used in admixtures should or can be explained fully or that the use of the admixtures in question should be postponed until these important theoretical explanations could be given. It took many years before the apparently simple reaction of portland cement and water alone—the setting and hardening of cement paste—could be explained theoretically, but high quality portland cement concrete was made long before these fundamental reactions were verified.

Various types of admixtures were used successfully for improving durability of concrete long before the advantages of air-entraining agents were discovered. With regard to increased strength, density and durability, those types of admixtures which reduce water-cement ratio and improve workability, were and are still being used most successfully. These admixtures work best if they also entrain a limited percentage of air.

The beneficial effect of pozzolanic admixtures to concrete has long been known but their use was considerably reduced because in the quantities considered necessary the fine pozzolanic admixtures greatly increased water-cement ratio and shrinkage of concrete. If pozzolans were used to replace a substantial part of cement there was also a substantial reduction in strength, especially in the first days, weeks or even months of hardening. After the last war various pozzolanic materials were used in many European countries mainly because portland cement was not readily available. Their use, either alone or combined with other admixtures, such as air-entraining agents, may improve durability.

*ACI Journal, Sept. 1950, *Proc.* V. 47, p. 26. Disc. 47-3 is a part of copyrighted Journal of the American Concrete Institute, V. 23, No. 4, Dec. 1951, *Proceedings* V. 47.

†Research Engineer and Consultant, London, England.

The author surveyed present-day views on air entrainment checked and supplemented by his own rich experience. On this subject the writer would like to stress a few points. The percentage of entrained air must be limited and controlled in the same way as was found necessary to limit and control water-cement ratio. Both limits have to be considered together and if results of special research are published, it should be possible to see in figures or diagrams the percentage of reduced water obtained by each percent of entrained air and also the percentages of cement and aggregate in absolute volumes. The author apparently does not fully agree with the view that the more air entrained the higher the resistance to frost. In the writer's opinion, too, "The average value is rather low in actual practice" is to be preferred.

The benefits of entrained air do not depend alone on the percentage of air in fresh and hardened concrete, but also on the size, shape, and distribution of air pores. Air bubbles entrained in fresh concrete differ from those resulting from evaporation of a considerable part of mixing water during hardening and drying of concrete. The latter are capillary pores absorbing water which may be very harmful under frost action if exceeding certain limits and especially if they are unfavorable in size, shape and distribution.

Some remarkable results were published in the ACI JOURNAL on the microscopical examination of the air pores, but the prospects indicated in the reply to the writer's discussion of a paper by Paul Klieger* that "Studies of the size and distribution of the air voids, of the saturation coefficient and their respective effects on resistance to freezing and thawing are under way in this and other laboratories" have not yet been satisfied. It is a very complex problem indeed, but it is considered to be the basis of testing the resistance to freezing and thawing both of natural stones and concrete. Special attention of Committee 212 is called to this saturation coefficient.

Discussion of a paper by Bruce E. Foster:

Use of Admixtures in Concrete Products†

By R. E. MADISON and AUTHOR

By R. E. MADISON‡

Under "water repelling agents" we can hardly agree with Mr. Foster's statement that "much of the water which penetrates walls built of block units passes through the joints." Our experience in the Detroit area indicates that the mortar joints are very considerably less subject to the passage of water than are the blocks. A great number of blocks examined and used in our laboratory were found so open-textured that water can almost be poured

*Klieger, Paul, "Effect of Entrained Air on Concrete made with So-called Sand-Gravel," Aggregates, ACI JOURNAL, Oct. 1948, *Proc.* V. 45, p. 149.
†ACI JOURNAL, Sept. 1950, *Proc.* V. 47, p. 32.
‡Technical Director, The Truscon Laboratories, Detroit, Mich.

through them. Thus with such large pore-openings to contend with, we might agree that waterproofing the blocks themselves might show but little benefit. On the other hand, walls made from these units must be treated to make them waterproof. Repeated field results have shown that this can be very effectively accomplished by thoroughly scrubbing into the wall surface a carefully selected cement base filler coat (cement base paint and fine sand). Another positive material which can be used alone or in conjunction with the filler coat is the newer type of iron base waterproofings which have a plastic consistency during application.

AUTHOR'S CLOSURE

Mr. Madison is correct in his statement that many concrete blocks offer little resistance to the passage of water and their surface must therefore be parged or otherwise treated to prevent water passage. The term "waterproofing" should perhaps have been written "integral waterproofing."

Discussion of a paper by Bruce E. Foster:

Use of Admixtures as Integral Waterproofing and Dampproofing Materials*

By R. E. MADISON and AUTHOR

By R. E. MADISON†

Mr. Foster's results on the soap or stearate type of waterproofing confirms our findings that their judicious use is beneficial in preparing concrete; however, we wish to set forth a different view on the definitions for "dampproofing" and "waterproofing."

Mr. Foster has stated that "even though the best concrete practices are adhered to, and the resulting concrete is impervious to water under pressure, passage of moisture due to capillary action might take place if the concrete is in contact with moisture or damp earth." According to his definition this concrete would not be dampproof but it would be waterproof. We submit that, in ordinary house construction if an additive agent accomplishes what he has thus suggested as required for dampproofing, and especially in the absence of any specific data obtained as a result of "sizeable pressures," the agent could properly be called a "waterproofing agent." Certainly there will be no exposure to sizeable pressures as apparently required under his definition of "waterproofing." If there is, his definition could be modified by referring to "pressure waterproofing." Since most home owners would refer to their basements as "waterproof" rather than "dampproof" if they are

*ACI JOURNAL, Sept. 1950, *Proc.* V. 47, p. 46.
†Technical Director, The Truscon Laboratories, Detroit, Mich.

dry, and since the conditions of "sizeable pressures" do not prevail, it is suggested that "waterproofing" in the case of home construction is a more apt term than "dampproofing." We concur heartily, of course, that admixtures are not substitutes for good materials, design and workmanship.

While we have not yet completed our tests on butyl stearate, we have a comment that should be of some interest. We have noted recently on the market certain material which contains in addition to other ingredients, both butyl alcohol and a stearate. Whether or not this combination is an attempt to indicate the presence of butyl stearate is not known but from a chemical standpoint it will be quickly recognized that such a combination is certainly not butyl stearate. In fact the inclusion of butyl alcohol might be entirely undesirable in a concrete mix.

With respect to Mr. Foster's conclusion on "finely subdivided dry materials" we do not check his results. Tests in our laboratory have shown that combinations can be used which will show a very considerable decrease in permeability when tested in concrete against plain mixes. These tests were performed under a method suggested by the Bureau of Standards and the test data has been submitted to the Bureau. Results indicated that permeability in specimens containing the admixtures was reduced in certain instances to as much as one-half of the permeability obtained in a plain mix.

AUTHOR'S CLOSURE

In discussing "waterproofing" integral admixtures a distinction between capillary transmission and passage of water due to external pressure is necessary. For this purpose the terms dampproofing and waterproofing were chosen and defined in accordance with paragraph 607a, p. 34, of the *Joint Committee's Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete* (1940). By dictionary* definition dampproofing appears to apply to vapor passage as well as liquid passage under low pressures, while a waterproof material is also a dampproof one. The terms were given a restricted definition in the paper and used simply because no better ones came to mind. It is felt that they are more descriptive than the terms "waterproof" and "pressure waterproof" suggested by Mr. Madison.

Interpretation of the mortar pot test mentioned by Mr. Madison is difficult. The make-up water required to maintain constant level in the pots is influenced by the absorption of the specimens, which are laboratory dry at the start of the test, by the transfer of moisture to the outside thru capillarity, and by seepage under the 0 to 9½ inches head of water in the pot. The results cannot therefore be taken strictly as a measure of capillarity, and an improvement noted upon the addition of a finely divided material might not be necessarily indicative of reduced capillarity.

We have found that the reproducibility of this test is poor and this finding is substantiated by some of Mr. Madison's data. We are inclined to the belief that a great deal more work would be required to demonstrate conclusively that finely divided fillers reduce capillarity.

*Webster's International Dictionary, 2nd Edition.

Simplicity and reliability are advantages of new sonic apparatus.

Improved Sonic Apparatus for Determining the Dynamic Modulus of Concrete Specimens*

By C. E. GOODELL†

SYNOPSIS

After a brief introduction to sonic testing and a description of commercial equipment, the apparatus built for the Michigan State Highway Department is discussed. Reliable results can be obtained by an unskilled worker with this compact equipment which has twice the frequency band spread of the usual oscillator. A wiring diagram of the sonic apparatus is included.

In recent years considerable emphasis has been placed on the theoretical aspects of the sonic method for testing materials and experience has proved it to be a satisfactory method for determining the dynamic modulus of a great many substances. Since it is a nondestructive and rapid method of testing, it's application and acceptance in the field of concrete testing has been widespread. Hornibrook[1] has shown this method to be ideally suited to freezing and thawing studies of concrete specimens where there exists a definite relationship between the decrease in modulus of elasticity and the modulus of rupture.

SONIC METHOD

The sonic method of testing is primarily one of determining the fundamental or natural frequency of vibration of the specimen under consideration, the dynamic modulus being related to this frequency by the following simple formula:[2]

$$E = \frac{4\pi^2 n^2 L^4 \rho}{r^2 k^4}$$

Where n = resonant frequency
L = length of specimen
ρ = mass density

*Received by the Institute Mar. 22, 1950. Title No. 47-4 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 22, No. 1, Sept. 1950, *Proceedings* V. 47. Separate prints are available at 35 cents each. **Discussion** (copies in triplicate) should reach the Institute not later than Jan. 1, 1951. Address 18263 W. McNichols, Detroit 19, Mich.

†Russell Mfg. Co., Caro, Mich. (formerly Physical Research Engineer, Highway Research Laboratory Michigan State Highway Dept.)

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r = radius of gyration
k = dimensional factor depending on mode of vibration and conditions of restraint.

In order to correct for the effects of shear, Goens[3] gives the formula above multiplied by a complicated correction factor which is a function of thickness, length and Poisson's ratio. Graphs for this correction factor are presented in the papers by Obert and Duvall[4] and Pickett.[2]

Many excellent papers have appeared on the theory of sonic testing,[1,2,3,4,5,6,7,8] therefore, the text of this paper will be confined to a discussion of the device developed to perform these tests.

All sonic test devices have the following basic units in common: (1) variable frequency audio oscillators; (2) specimen vibration mechanism, or driving unit; (3) pick-up devices; and (4) resonance indicator. These units function as follows: The oscillator actuates the driving mechanism at any desired frequency within the range of the apparatus. The driving mechanism vibrates the specimen. The signal generated by the oscillator is fed into the resonance indicator, and this results in a certain operating level indication on the resonance meter, or an amplitude indication on a cathode ray tube. The frequency at which the specimen is vibrated is transmitted to the resonance indicator through a pick-up device, usually of the crystal type. This signal adds to the oscillator signal to raise the operating level of the resonance indicator. Since the electrical output of the pick-up increases with mechanical input to the crystal, and the mechanical input is maximum when a specimen is vibrating at its fundamental frequency, then the resonance indicating meter will give its greatest reading, or a cathode ray tube will register greatest amplitude, when the specimen is vibrating at its fundamental frequency.

When the indicator incorporates a cathode ray tube an additional and more reliable principle of determining resonance is used. The oscillator signal is fed to one set of plates on the tube and the amplified pick-up signal to the other set of plates. When these signals are of the same frequency and in phase a Lissajous circle is seen on the tube screen.

Fig. 1 illustrates, in block diagram form, the essential elements, and their interconnections, for use in testing concrete specimens.

APPARATUS

Most commercial audio oscillators are of the beat frequency or R-C type, neither of which is particularly well suited to the method of sonic testing. If the oscillator is of the beat-frequency type it is subject to wide frequency drifts with variations of time and temperature and must be continually recalibrated if accurate results are to be expected. If the oscillator is of the R-C or Wein bridge type it is less subject to

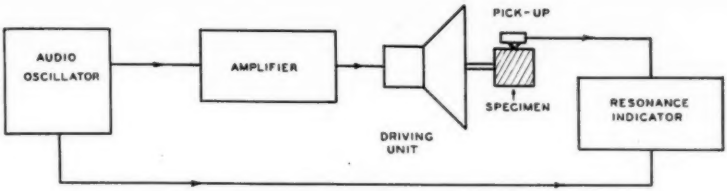


Fig. 1—Block diagram of sonic apparatus

frequency variations but the scale, in general, is too crowded to permit an accurate determination of the frequency setting. This objection can be overcome by the use of commercial interpolation oscillators but their added expense can seldom be justified.

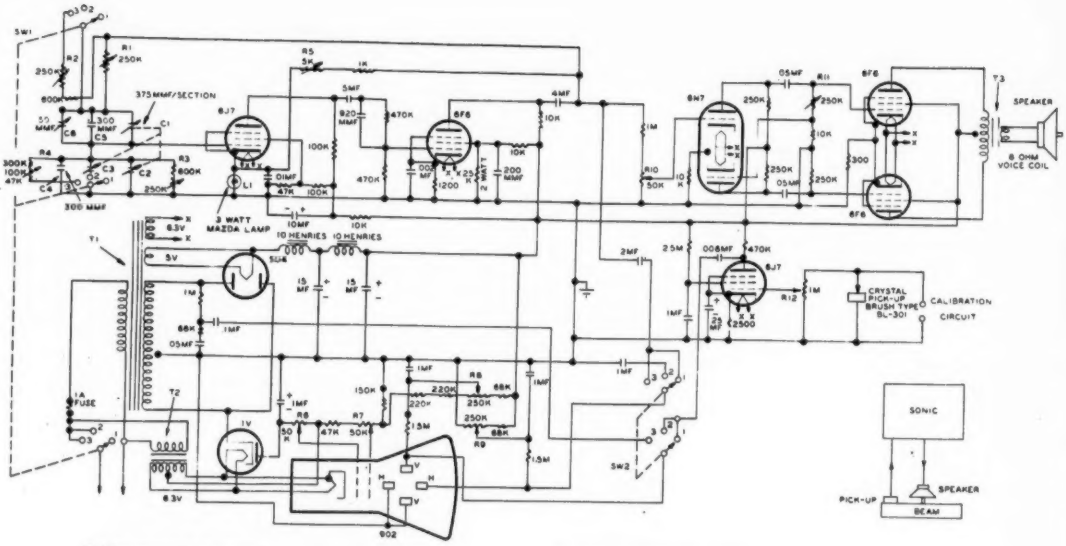
The resonance indicator can be a simple vacuum tube voltmeter or a commercial cathode ray oscilloscope. If a vacuum tube voltmeter is employed the resonant point is apt to be determined less accurately than with a cathode ray oscilloscope and in addition it is extremely difficult to differentiate between the fundamental resonant frequency and one of the many harmonics that are present. By means of Lissajous figures on the cathode ray oscilloscope this latter objection can be eliminated, but the purchase of a commercial oscilloscope can hardly be justified to perform this one simple task.

IMPROVED COMPACT UNIT

The sonic apparatus developed for the Michigan State Highway Department combines all of these basic units into a compact unit that is simple enough in operation for the most unskilled worker, reliable, and of sufficient accuracy for the most critical tests. In addition, it possesses certain technical advantages unobtainable with standard commercial equipment.

Fig. 2 shows the circuit diagram of the sonic apparatus developed to combine all of the best features possible and to eliminate the primary objections of most commercially available equipment. A parts list is given in Table 1.

The band spread on the frequency dial is at least twice that of the ordinary oscillator, yet the oscillator stability is remarkably good. Tests with this apparatus have shown that the frequency drift is less than 0.5 percent per hour, after the first ten minutes of operation. The output wave shape shows little distortion and the amplitude is practically constant over the entire frequency range for any one setting of the amplitude control.



R1,R3 - LOW FREQ CALIB CONTROLS
R2,R4 - HIGH FREQ CALIB CONTROLS
R5 - FEEDBACK CONTROL
R6 - C.R.O. INTENSITY CONTROL
R7 - C.R.O. FOCUS CONTROL
R8 - C.R.O. VERTICAL CENTERING
R9 - C.R.O. HORIZONTAL CENTERING
R10 - VOLUME CONTROL
R11 - PHASE INVERTER BALANCE
R12 - SENSITIVITY CONTROL

SW1
POS. NO. 1 - OFF
POS. NO. 2 - LOW FREQ. SCALE, 600-1500 CPS
POS. NO. 3 - HIGH FREQ. SCALE, 1400-3000 CPS

SW2
POS. NO. 1 - LISSAJOUS RESONANCE INDICATOR
POS. NO. 2 - AMPLITUDE OF BEAM VIBRATION
POS. NO. 3 - OSCILLATOR CALIBRATION

T1 - THORDARSON POWER TRANS. T22R07
700V CT, 200MA, 5V AT 3A, 6.3V AT 6A
T2 - THORDARSON FILAMENT TRANS. T21F10
6.3V CT AT 3A
T3 - THORDARSON OUTPUT TRANS. T22S84
PRIMARY - 10,000Ω CT, SECONDARY -
3.2-4, 6-8, 15, 250, 500 OHMS

Fig. 2—Circuit diagram of sonic apparatus

TABLE 1—SONIC APPARATUS PARTS LIST

Resistors*	Potentiometers†	Condensers‡
1—300	1—5K	1—50 mmf air padder
1—1000	3—50K	1—3 gang, 375 mmf per section, variable cond.
1—1200	1—100K	1—200 mmf
1—2500	6—250K	2—300 mmf
5—10K	1—1M	1—920 mmf
1—25K, 2 watt		1—0.002
3—47K		1—0.006
3—68K		1—0.01
2—100K		3—0.05
1—150K		5—0.10
2—220K		1—0.20
3—250K		1—1.0, 600 V
1—300K		1—4.0
3—470K		1—10, 450 V, electrolytic
2—600K		2—15, 450 V, electrolytic
2—1M		1—25, 25 V, electrolytic
2—1.5M		
1—2.5M		

Vacuum tubes	Miscellaneous
2—6J7	1—Mazda lamp, 3 watt, 120 V
3—6F6	2—Filter choke, 10 henry, 250 ohm
1—6N7	1—Power transformer, 700 V ct, 200 ma; 5 V at 3 amp; 6.3 V at 6 amp
1—1V	
1—902	1—Filament transformer, 6.3 V at 3 amp
	1—Output transformer, primary 10,000 ohms ct; secondary 3.2-4, 6-8, 15, 250, 500 ohms
	1—Selector switch, 3 pole, 3 position
	1—Selector switch, 2 pole, 3 position

*All resistors 1 watt, ± 10 percent tolerance unless otherwise specified. All resistances in ohms unless otherwise specified. K = × 1000; M = × 1,000,000.
†All potentiometers 1 watt.
‡All condensers 400 V rating unless otherwise specified. All capacitances in microfarads unless otherwise specified.

Fig. 2—Circuit diagram of sonic apparatus

The increased band spread has been obtained by properly proportioning the amount of fixed and variable capacitances in the frequency-determining network consisting of $R_1,R_3,C_1,C_2,C_4,C_5,C_6$ for the low frequency range and $R_1,R_2,R_3,R_4,C_1,C_2,C_3,C_4,C_5,C_6$ for the high frequency range (See Fig. 2).

Frequency drift is inherently low in a Wein bridge type of oscillator since the frequency-determining elements are required to pass only minute currents and the power dissipated by these elements is, therefore, correspondingly small. Thus, changes in circuit element values are caused almost entirely by changes in ambient temperature. With proper ventilation and placement of parts the frequency drift can be kept small.

Constant amplitude and low distortion of the oscillator output wave is accomplished by the negative feedback path consisting of elements R_5, L_1. The 3-watt Mazda lamp L_1 serves as a nonlinear circuit element which automatically adjusts the feedback and amplitude to the proper level.

The 2-in. cathode ray tube serves three purposes. It is used for checking the calibration of the oscillator, the 60-cycle power frequency being used as a standard. It is also used as a resonance indicator, making use of Lissajous figures to ascertain the fundamental frequency, and in addition it is used to indicate the amplitude of the resonant peak when comparative tests are required.

Fig. 3 illustrates typical Lissajous figures that may be observed on the cathode ray tube during the testing of a specimen. Fig. 3a represents the case where the frequency of vibration of the bar is equal to the applied frequency; and Fig. 3b and 3c illustrate the case when the frequencies of vibration of the specimen are two and three times, respectively, the applied frequency. Fig. 4 is a photograph of the Lissajous figure observed during the testing of a specimen.

Calibration controls for the oscillator are included within the cabinet to permit recalibration of the oscillator in the event changes in the electrical components should alter the frequency scales. In addition, the usual centering, focusing and intensity controls are included for the cathode ray tube. These controls are adjusted and then locked, as further adjustment is not necessary unless the tube is replaced.

Fig. 5 shows the sonic apparatus together with the specimen support and driving mechanism. The specimen is supported on knife edges at the nodal points of the bar and the entire assembly mounted upon a moveable carriage. A hand wheel moves the carriage, support and specimen until the specimen makes the proper contact with the driving unit. The crystal acceleration type pick-up is spring suspended from a moveable rod that can be rotated so as to place the pick-up on the specimen during test, or rotated out of the field while changing specimens. There is evidence to indicate that a magnetic velocity type pick-up might be better suited to this application than the crystal type originally employed. Crystal pick-ups are basically acceleration devices and therefore tend to amplify the harmonics of a wave more than the fundamental frequency. This can cause considerable confusion

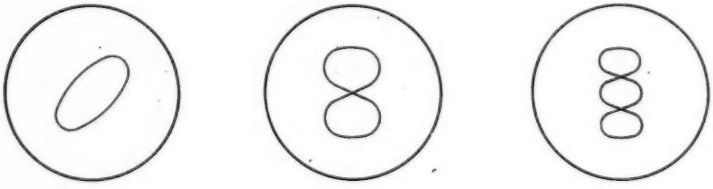


Fig. 3—Characteristic Lissajous figures

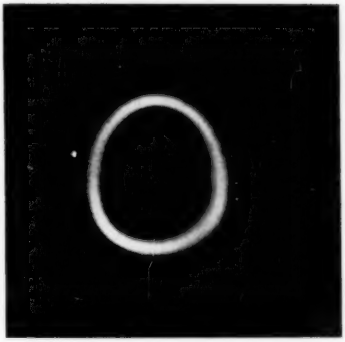


Fig. 4—Lissajous figure obtained during test when beam is vibrating at its fundamental frequency

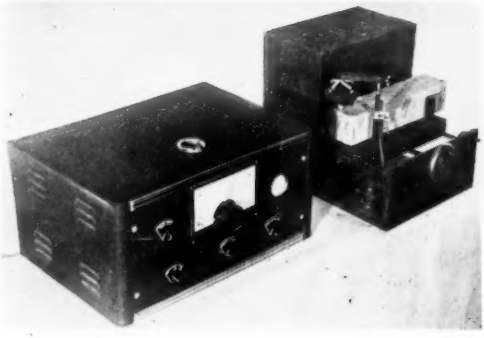


Fig. 5—Sonic apparatus ready for test with beam mounted on nodal supports and crystal pick-up in position

when trying to determine the fundamental frequency of vibration of the specimen under test.

This sonic apparatus has superseded an earlier device used for sonic testing and the present indications are that it is far superior in every respect.

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A practical and more rigorous method is presented to replace present day approximations.

Analysis of Three-Dimensional Beam-and-Girder Framing*

By PHIL M. FERGUSON†

SYNOPSIS

The beam-and-girder floor with some beams carried directly by columns and others supported on girders is cited as a practical problem in frame analysis that must include the torsional stiffness of the girder. Curves showing how moment coefficients vary with this torsional stiffness are developed for a few simple cases in interior panels. These show the weakness of rule-of-thumb methods.

A practical calculation form is set up for use with the moment distribution method in solving three-dimensional problems of this type.

INTRODUCTION

Although many problems in building frame analysis require consideration of torsional stiffness for correct results, it has been common practice to ignore this complicating factor. Sometimes the torsional stiffness has been completely ignored; at other times arbitrary or rule-of-thumb approximations have been introduced. Because such problems are three dimensional rather than planar, some have erroneously thought that better solutions must necessarily be unwieldly.

A common problem is that of beam-and-girder floors. Here some beams are carried directly by columns and others are supported by girders which in turn frame into the columns. There is a considerable difference in the joint restraints provided for the two types of beams and considerable difference in the maximum moments acting on the beams. To evaluate these maximum moments, the torsional stiffness of the girder must be introduced into the analysis.

The Joint Committee Specification (Par. 808 b) suggests that the beams supported by girders may be analyzed as though supported by

*Received by the Institute Nov. 10, 1949. Title No. 47-5 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 22 No. 1, Sept. 1950, *Proceedings* V. 47. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Jan. 1, 1951. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Chairman, Dept. of Civil Engineering, University of Texas, Austin, Texas.

columns having half the average stiffness of the adjoining beams. Presumably this implies that such beams and girders will be ignored in analyzing the beams that frame directly into the columns; and the designer is left to his own discretion in evaluating the moments acting on the column due to unbalanced loading.

The ACI Building Code offers no suggestions for such problems except to state that the design moments assigned to the columns must be consistent with those assumed in the analysis of the beams.

It has been suggested elsewhere* that beams framing into girders and beams framing into columns be made alike, with their analysis based on the use of "floor stiffness" instead of beam stiffness. The floor stiffness would be the total stiffness of the several parallel beams in a panel. Thus if each girder carried a single beam at its mid-point, the floor stiffness would be twice the stiffness of a single beam.

TORSIONAL STIFFNESS

Usable methods of analysis by moment distribution have been developed† that include torsional resistance of girders. They are recommended for general usage and some columnar calculation forms that have facilitated such analyses are presented herein.

The one supplement to ordinary moment distribution that is needed is a relationship between the applied torque T and the resulting angle of rotation Θ. The usual formula for shafts can be used with slight modifications, $\Theta = TL/JG$, where

Θ = angle of rotation in length L
T = torque acting
G = shearing modulus of elasticity $\doteq 0.4E$
J = torsional stiffness factor (taking the place of the polar moment of inertia used with circular sections).

Methods of calculating J for T-shaped members are given in Report 334 of the National Advisory Committee for Aeronautics, by Thayer and March.‡ The torsional stiffness can be defined as

$$K_g = T/\Theta = JG/L \quad \text{or} \quad T = K_g\Theta$$

This K_g is directly comparable to the bending stiffness $4EI/L$ as commonly used in moment distribution. Although the term K_b is often used to represent the entire product $4EI/L$ shown here, in this paper it will hereafter be used to represent only EI/L, such that when the far end of a beam is fixed,

$$M = 4K_b\Theta$$

where Θ = end deflection angle due to an applied bending moment M.

*_Continuity in Concrete Building Frames_, Portland Cement Assn., 3rd Ed., p. 33.
†_Reinforced Concrete Structures_, Peabody, 2nd Ed., pp. 412-420.
‡The earlier reference to Peabody may be a more convenient reference.

In order to indicate the effect of torsional stiffness and at the same time to develop specific data which is of interest, several common cases are analyzed here. Algebraic solutions have been obtained, using the slope deflection approach with certain rotation angles as unknowns.

ANALYSIS

Positive live load moments on interior beams—two beams per panel

When beams frame midway between columns as in Fig. 1, the same beam size is commonly used for both sets of beams. Analysis proceeds on the assumption that columns are fixed at the floors above and below. The loading for maximum positive moment, which also gives maximum moment on the columns, includes live load on spans AA' and BB' as well as alternate spans beyond AB and $A'B'$. Joint rotations occur about the x-x axis, but none about the y-y axis. The structure and loading may be considered as symmetrical about the centerline of $A'A$ and AA'', where A'' is beyond y. Then $\Theta_A = -\Theta_{A'}$ and $\Theta_B = -\Theta_{B'}$. The vertical deflection of B and B' will be equal and this will not influence the beam moments. It is noted that torques develop in the girder, but that the amount depends upon the difference between Θ_B and Θ_A. Then

T = torque on girder AB (positive when resisting Θ_B)
$= K_g (\Theta_B - \Theta_A)$
where $K_g = JG/L$ for length AB of girder
Let $K_b = EI/L$ of beam
$K_c = EI/L$ of column
w = *live* load on beam, lb per lin. ft

$$\Sigma M_x \text{ at joint } A = 0 = -1/12\, wL^2 + 2K_b\Theta_A \text{ (for } AA', \text{ since } \Theta_{A'} = -\Theta_A) + 2K_b\Theta_A \text{ (for } AA'') + 4K_c\Theta_A + 4K_c\Theta_A - 2T$$
$$= -1/12\, wL^2 + (4K_b + 8K_c)\,\Theta_A - 2(\Theta_B - \Theta_A)K_g = 0 \quad \ldots\ldots\ldots\ldots (1)$$

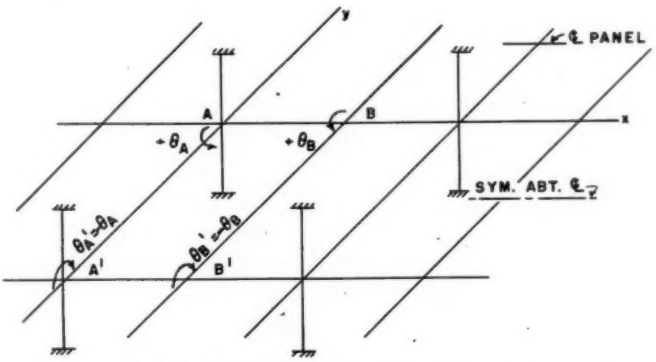


Fig. 1—Typical interior panel, equal spans, loaded for positive moment

$$\Sigma M_x \text{ at joint } B = 0 = -1/12\ wL^2 + 2K_b\Theta_B + 2K_b\Theta_B + 2T$$
$$= -1/12\ wL^2 + 4K_b\Theta_B + 2(\Theta_B - \Theta_A)K_g = 0 \quad \ldots\ldots (2)$$

These equations give:

$$\Theta_A = 1/12\ wL^2 \times \frac{K_b + K_g}{4K_bK_g + 8K_bK_c + 4K_b^2 + 4K_cK_g}$$

$$\Theta_B = 1/12\ wL^2 \times \frac{2K_c + K_b + K_g}{4K_bK_g + 8K_bK_c + 4K_b^2 + 4K_cK_g}$$

$$M_{AA'} = -1/12\ wL^2 + 2K_b\Theta_A = -1/12\ wL^2 + 1/12\ wL^2 \times \frac{2K_b(K_b + K_g)}{4K_bK_g + 8K_bK_c + 4K_b^2 + 4K_cK_g} \quad \ldots\ldots (3)$$

$$\text{Pos. } M_{AA'} = 1/8\ wL^2 + M_{AA'} = 1/24\ wL^2 + 1/12\ wL^2 \times \frac{(K_b/K_g) + 1}{2 + 4(K_c/K_g) + 2(K_b/K_g) + 2(K_c/K_b)} \quad \ldots\ldots (4)$$

$$M_{BB'} = -1/12\ wL^2 + 2K_b\Theta_B = -1/12\ wL^2 + 1/12\ wL^2 \times \frac{2K_b(2K_c + K_b + K_g)}{4K_bK_g + 8K_bK_c + 4K_b^2 + 4K_cK_g} \quad \ldots\ldots (5)$$

$$\text{Pos. } M_{BB'} = 1/8\ wL^2 + M_{BB'} = 1/24\ wL^2 + 1/12\ wL^2 \times \frac{2(K_c/K_g) + (K_b/K_g) + 1}{2 + 4(K_c/K_g) + 2(K_b/K_g) + 2(K_c/K_b)}$$
$$= 1/12\ wL^2 - 1/12\ wL^2 \times \frac{K_c/K_b}{2 + 4(K_c/K_g) + 2(K_b/K_g) + 2(K_c/K_b)} \quad \ldots\ldots (6)$$

Values of the coefficients of wL^2 for $+M_{AA'}$ and $+M_{BB'}$ are plotted in Fig. 2 for a considerable range of stiffness values. Curves for $K_b/K_g = \infty$ corespond to zero girder stiffness and independent action of beams AA' and BB'. The one curve for $K_b/K_g = 0$ corresponds to a girder of infinite stiffness, in which case both beams have identical moments. However, since values of K_b/K_g around 0.5 and 1.0 seem to be most common, it will be noted that the two beams develop widely different moments.

A comparison with the approximate analysis mentioned above is in order. The dashed line marked "J.C." represents $+M_{BB'}$ under the Joint Committee suggestion, and presumably this would go with $M_{AA'}$ for $K_b/K_g = \infty$. Thus the Joint Committee suggestion leads to values of $+M_{AA'}$ that are a little low, while $+M_{BB'}$ would be quite low unless columns were very stiff. The use of a "floor stiffness" in the analysis would give the curve shown for $K_b/K_g = 0$, which is satisfactory for $+M_{AA'}$ but entirely too low for $+M_{BB'}$.

It should be noted that dead load moments are not included in the above. For this simple case, dead load moments are simply those for fixed ends, that is, $+M = +wL^2/24$.

Negative moments on interior beams—two beams per panel

Maximum negative moments at A and B in the frame of Fig. 3 occur when the loading is symmetrically placed each side of girder AB, with

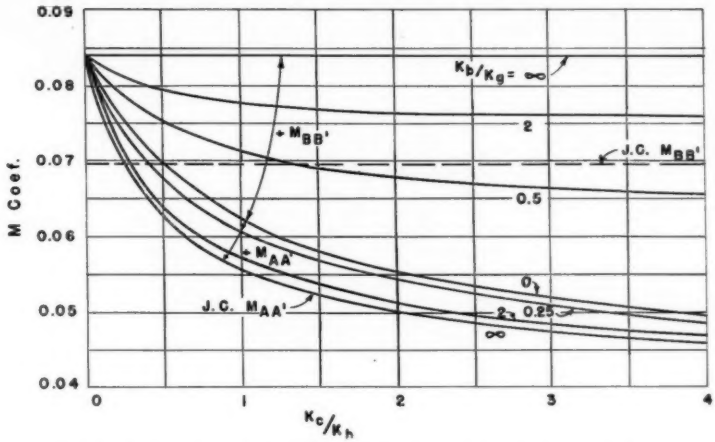


Fig. 2—Positive moment coefficients for live load on interior beams of Fig. 1

BB' and AA' loaded, $B'B''$ and $A'A''$ unloaded, and with the span beyond B'' and A'' loaded. For analysis, symmetry about AB will be represented by fixed ends at A and B; and Θ_B'' will be assumed approximately equal and opposite in sign to Θ_B'. A like relation holds between Θ_A'' and Θ_A'. Rotations at joints B' and A' can be analyzed about as in the case for positive beam moments. The fact that B deflects vertically more than B' and B'' will be neglected. The carry-over moments from B' to B and A' to A must be combined with the original fixed-end moments at A and B to give the maximum negative moments desired.

$$\Sigma M_z \text{ at joint } A' = 0 = -1/12\ wL^2 + 2K_b\Theta_{A'} \text{ (for } A'A'') + 4K_b\Theta_{A'} \text{ (for } A'A) + 4K_c\Theta_{A'} + 4K_c\Theta_{A'} - 2T = -1/12\ wL^2 + (6K_b + 8K_c)\Theta_{A'} - 2(\Theta_{B'} - \Theta_{A'})K_g = 0 \quad (7)$$

$$\Sigma M_z \text{ at joint } B' = 0 = -1/12\ wL^2 + 2K_b\Theta_{B'} + 4K_b\Theta_{B'} + 2T = -1/12\ wL^2 + 6K_b\Theta_{B'} + 2(\Theta_{B'} - \Theta_{A'})K_g = 0 \quad (8)$$

These equations give:

$$\Theta_{A'} = +1/12\ wL^2 \times \frac{3K_b + 2K_g}{18K_b^2 + 12K_bK_g + 24K_bK_c + 8K_cK_g}$$

$$\Theta_{B'} = +1/12\ wL^2 \times \frac{3K_b + 4K_c + 2K_g}{18K_b^2 + 12K_bK_g + 24K_bK_c + 8K_cK_g}$$

These rotations increase the negative moments at A and B by $2K_b\Theta_{A'}$ and $2K_b\Theta_{B'}$:

$$M_{AA'} = -1/12\ wL^2 - 1/12\ wL^2 \times \frac{K_b\,(3K_b + 2K_g)}{9K_b^2 + 6K_bK_g + 12K_bK_c + 4K_cK_g}$$

$$= -1/12\ wL^2 - 1/12\ wL^2 \times \frac{K_b/K_g + 0.667}{3K_b/K_g + 2 + 4K_c/K_g + 1.333K_c/K_b} \quad (9)$$

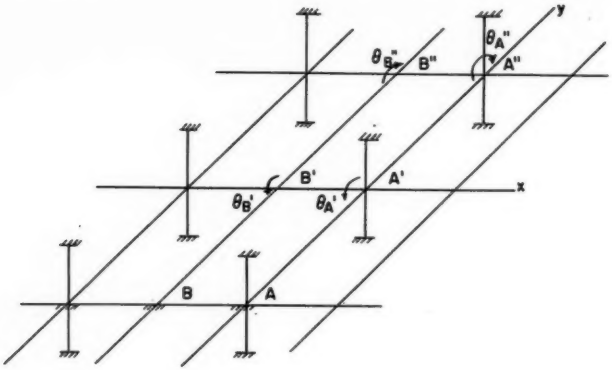


Fig. 3—Typical interior panels, equal spans, loaded for negative moment at A and B

$$M_{BB'} = -1/12\ wL^2 - 1/12\ wL^2 \times \frac{K_b(3K_b + 4K_c + 2K_c)}{9K_b^2 + 6K_bK_g + 12K_bK_c + 4K_cK_g}$$

$$= -1/12\ wL^2 - 1/12\ wL^2 \times \frac{K_b/K_g + 1.333K_c/K_g + 0.667}{3K_b/K_g + 2 + 4K_c/K_g + 1.333K_c/K_b} \quad (10)$$

These values of coefficients for $-M_{AA'}$ and $-M_{BB'}$ are plotted as solid line curves in Fig. 4. The Joint Committee recommendation for $M_{BB'}$ is also shown. Presumably this would be used with the lowest curve for $M_{AA'}$. The use of the entire floor stiffness would lead to the curve for $K_b/K_g = 0$.

Three beams per panel—any symmetrical layout

Although usually three beams per panel implies beams at the third points of the girder, this analysis applies to any case of two beams symmetrically placed between the columns. The analysis follows the same steps as that for two beams per panel, the sole difference being that the symmetrical beam layout indicates that there can be no torsion on the central part of the girder. Hence ΣM_X at joint B includes only T instead of $2T'$. This gives for the maximum positive moments:

$$\text{Pos. } M_{BB'} = +1/12\ wL^2 - 1/12\ wL^2 \times \frac{K_c/K_b}{3 + 2K_c/K_b + 8K_c/K_g + 4K_b/K_g} \quad (11)$$

$$\text{Pos. } M_{AA'} = +1/24\ wL^2 + 1/24\ wL^2 \times \frac{1.5 + 2K_b/K_g}{1.5 + K_c/K_b + 4K_c/K_g + 2K_b/K_g} \quad (12)$$

Similarly, maximum negative moments become:

$$M_{AA'} = -1/12\ wL^2 - 1/12\ wL^2 \times \frac{0.5 + K_b/K_g}{1.5 + 0.667\ K_c/K_b + 4K_c/K_g + 3K_b/K_c} \quad (13)$$

$$M_{BB'} = -1/9\ wL^2 - 1/54\ wL^2 \times \frac{K_c/K_b}{1.5 + 0.667K_c/K_b + 4K_c/K_g + 3K_b/K_g} \quad (14)$$

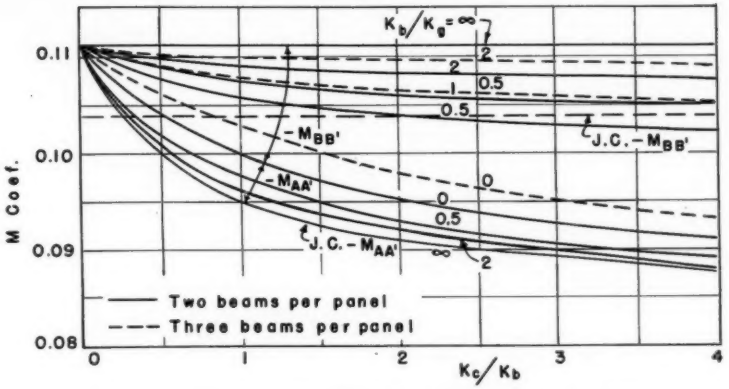


Fig. 4—Negative moment coefficients for live load on interior beams

Curves for the positive moment coefficients are shown in Fig. 5. It will be noted that approximate methods are more in error for this case than where there are two beams per panel (Fig. 2).

The negative moment coefficients for $M_{BB'}$ are shown in dotted lines on Fig. 4 because they are not greatly different from those for two beams per panel. Except for $K_b/K_g = 0$, the curves for $M_{AA'}$ fall so close to the corresponding curves for two beams per panel that they can not conveniently be shown.

Column moments

Maximum moments in interior columns are given by the same loading that gives maximum positive beam moments. Hence the values of Θ_A already found can be used directly to give column moments:

$$M_c = 4K_c\Theta_A$$

For two beams per bay, this becomes:

$$M_c = 1/12\ wL^2 \times \frac{4K_c^2\,(K_b + K_g)}{4K_bK_g + 8K_bK_c + 4K_b^2 + 4K_cK_g}$$

$$= 1/12\ wL^2 \times \frac{(K_c/K_b)\,(K_b/K_g + 1)}{1 + 2(K_c/K_g) + K_b/K_g + K_c/K_b} \qquad (15)$$

These coefficients are shown as solid line curves in Fig. 6. The top curve corresponds to both beams rigidly connected to the column; the lowest curve considers BB' as having no influence upon the column moment, either as to load or stiffness.

For three beams per bay, the column moment becomes:

$$M_c = 1/12\ wL^2 \times \frac{2K_c/K_g + 1.5K_c/K_b}{1.5 + K_c/K_b + 4(K_c/K_g) + 2(K_b/K_g)} \qquad (16)$$

These coefficients are plotted with dotted lines in Fig. 6 for low values of K_b/K_g. For $K_b/K_g = 2$ or more, the values are never more than 2 percent different from those for two beams per panel.

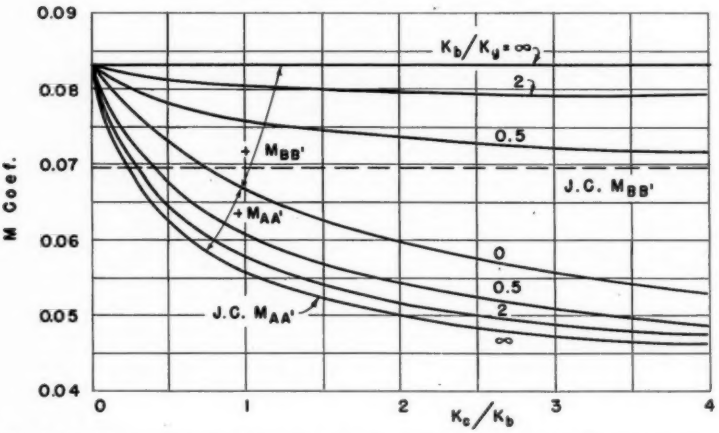


Fig. 5—Positive moment coefficients for live load on interior beams, three beams per panel, equal spans

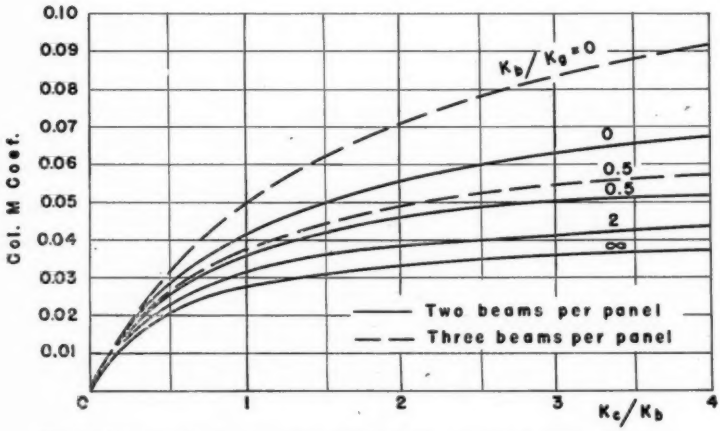


Fig. 6—Interior column moment coefficients for live load only, equal beam spans

Exterior panels

Exterior panels do not lend themselves to as simple an algebraic analysis as interior panels. A study on a limited basis has indicated that approximate methods produce about the same percentage errors in exterior as in interior panels.

GENERAL CONCLUSIONS ON INTERIOR PANEL BEAM AND COLUMN MOMENTS

The above analyses give the following indications for live load moments. Dead load moments are the same for all beams when spans are equal.

Positive moments

1. In general, positive live load moments are apt to be from 15 to 35 percent larger on beams framing into girders than on beams framing into columns.

2. The Joint Committee suggestion for beams framing into girders gives moments slightly low for beams framing into the center of a girder and seriously low for beams framing at the third points of the girder.

3. For beams that frame directly into the columns, the live load positive moment is larger than that obtained by ignoring the other beams and smaller than that obtained by using the full panel stiffness. The difference in these two methods is in the order of $0.006\ wL^2$ for two beams per panel and about $0.010\ wL^2$ for three beams per panel.

Negative moments

1. Negative live load moments are apt to be from 10 to 15 percent larger in beams framing into girders than for those framing into columns.

2. The Joint Committee proposal for beams framing into girders is on the unsafe side.

3. It would be safe, and not very wasteful, to calculate moments for beams framing into columns using the full panel stiffness, while for beams framing into girders knife edge supports could be assumed.

Columns

1. Column live load moments are definitely much more than would be brought in by a single beam, in the order of 20 to 40 percent more; and less than would be given by full panel load and stiffness, by about the same percentages.

2. For usual values of K_b/K_g the coefficients are only slightly different for three beams per panel than for two beams per panel.

CALCULATIONS BY MOMENT DISTRIBUTION

A study like that above indicates that no rule-of-thumb method is actually adequate to care for the simple cases presented. It is obvious

that engineers should become more familiar with moment distribution as applied to three-dimensional frames, both to handle more complex cases and to build up their judgment in handling ordinary problems.

The necessary expansion of methods used for planar frames is quite simple, especially where analysis can be limited to one floor level plus the columns above and below. The fundamental ideas have been explained elsewhere* and a satisfactory tabulation form is presented in Fig. 7. This is an expansion of the standard Portland Cement Assn. form and is applied directly upon a *plan* view of the frame. Its use will be explained by a sample problem.

EXAMPLE

Find the maximum positive live load moments on the beams of an exterior panel (Fig. 7) for equal spans and relative stiffness values as follows:

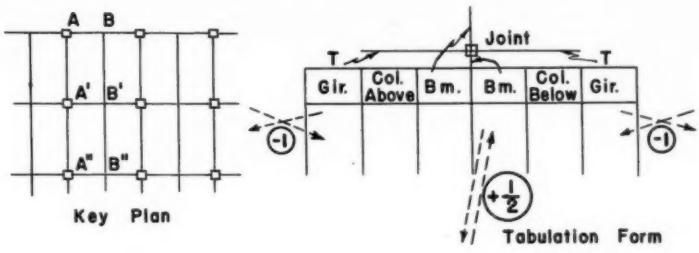
	Exterior panel		Interior panel	
Beams	AA' and BB',	$4EI/L = 1$	$A'A''$ and $B'B''$	$= 2EI/L = 0.5$
Columns	At A,	$4EI/L = 0.6$	At A',	$4EI/L = 1$
Girders (half length)	AB (torsion),	$JG/L = 0.6$	$A'B'$ (torsion),	$JG/L = 1$

The exterior spans and the second interior spans of the beams will be loaded while the first interior spans will be unloaded. This approximate symmetry about the middle of the first interior span permits the use of these beams in the analysis as terminal members with an effective stiffness of $2EI/L$, as tabulated in the above table for $A'A''$ and $B'B''$. Columns are taken as fixed at floors above and below, although other conditions of restraint can be represented by modified stiffness values. The difference in vertical deflection of B' as compared to B is ignored.

The calculation form in Fig. 7 is almost self explanatory. A torque or moment that tends to rotate a joint clockwise is taken as positive. Where torques and moments act about axis x-x, as in Fig. 7, the signs are fixed by viewing the joints from the right margin of the page. It is convenient to mark a T over columns of data that represent torques. This helps a person to remember that the carry-over-factor for torque is different from that for moment, the torque carry-over-factor being *minus* one for this sign convention.

Only four joints, representing half of each girder, are shown, because further joints would be exact repetitions. Under these conditions equal torques are always carried into a joint from each side. The fixed-end moments have each been taken as 100 for this example in the absence of definite loads and spans. The final results will thus give moments as a percentage of the fixed-end moments. Distributions have been alter-

*Reinforced Concrete Structures, Peabody, 2nd Ed., pp. 412-420.



A

	T Gir.	Col. Above	AA'	Col. Below	T AB
Stiff.	0.6	0.6	1.0	0.6	0.6
D.F.	0.18⁻	0.18⁻	0.29	0.18⁻	0.18⁻
MF			-100		
D	+18	+17	+29	+18	+18
C	-40		-16		-41
D	+17	+18	+28	+17	+17
C	-13		-6		-13
D	+6	+6	+9	+5	+6
C	-5		-2		-5
D	+2	+2	+3	+3	+2
D	-1		-1		-2
C	0	+1	+1	+1	+1
Σ	(-16)	(+44)	-55	(+44)	(-17)

B

	T BA	BB'	T Gir.
	0.6	1.0	0.6
	0.27	0.46	0.27
MF		-100	
C	-18	-14	-18
D	+41	+69	+40
	-17	-14	-17
	+13	+22	+13
	-6	-5	-6
	+5	+7	+5
	-2	-2	-2
	+2	+3	+1
	-1	-1	0
	0	+1	+1
Σ	(+17)	-34	(+17)

A'

	T Gir.	Col. Above	A'A	A'A"	Col. Below	T A'B'
	1.0	1.0	1.0	0.5	1.0	1.0
	0.18+	0.18+	0.18+	0.09	0.18+	0.18+
MF			+100			
C	+29		+14			+29
D	-31	-31	-31	-16	-32	-31
C	+27		+14			+28
D	-13	-13	-12	-6	-12	-13
	+11		+4			+11
	-5	-4	-5	-2	-5	-5
	+4		+2			+4
	-2	-2	-2	-1	-1	-2
	+2		0			+1
	0	-1	-1	0	-1	0
Σ	(+22)	(-51)	+83	(-25)	(-51)	(+22)

A"

B'

	T B'A'	B'B	B'B"	T Gir.
Stiff.	1.0	1.0	0.5	1.0
D.F.	0.29⁻	0.29⁻	0.14	0.29⁻
MF		+100		
D	-29	-28	-14	-29
C	+31	+34		+31
D	-28	-28	-13	-27
C	+13	+11		+13
D	-11	-10	-5	-11
C	+5	+4		+5
D	-4	-4	-2	-4
C	+2	+2		+2
D	-1	-2	-1	-2
Σ	(-22)	+79	(-35)	(-22)

B"

Fig. 7—Moment distribution calculations for moments on exterior beams. Note that data is arranged on plan view of floor

nated between joints A-B' and joints A'-B and have been carried a cycle or two further than really necessary.

The positive moments at the centerline of beams become, for uniform loads:

$$\text{Pos. } M_{AA'} = 1/8\ wL^2 - \left(\frac{0.55 + 0.83}{2}\right) 1/12\ wL^2 = (0.125 - 0.057)\ wL^2$$

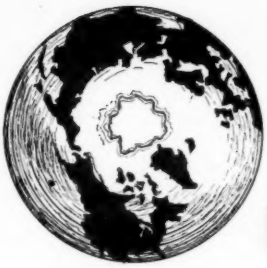
$$= + 0.068\ wL^2$$

$$\text{Pos. } M_{BB'} = 1/8\ wL^2 - \left(\frac{0.34 + 0.79}{2}\right) 1/12\ wL^2 = (0.125 - 0.047)\ wL^2$$

$$= + 0.078\ wL^2$$

A similar calculation form can be used for problems involving rotations about the y-y axis, but it should be obvious that the vertical movement of joints B and B' then becomes a major part of the problem. Since this deflection problem involves all the complications inherent in the usual sidesway problem, it is fortunate that many problems can be solved using only the moments about the x-x axis.

The whole problem of end restraints on exterior spans and the resulting torques on spandrel beams and girders, as well as the resulting bending moments on exterior columns, needs more study. Some of these problems can be solved for specific cases with the methods here presented.



Letters from Readers*

BY WAY OF SYNOPSIS

E. H. LICHTENBERG comments on the evolution of consistency meters.

Will fly ash in concrete affect the corrosion of metallic members asks a READER.

A. C. KAHL proposes some refinements in the methods for determining air content of concrete that will eliminate the difficulties caused by sampling errors.

Information is lacking on many properties of Shotcrete states R. A. SPENCER.

M. HIRSCHTHAL suggests that air-entraining concrete be used only for the exposed portions of a structure.

A READER discusses the problems of building an intake structure in the arctic.

Lt. Col. W. B. SYKES describes the manufacture of prestressed concrete railroad ties.

Supplemental information is presented by ISSER HARBAND on the problem of a paneled ceiling loaded by concentrated loads at the intersection of the beams.

Consistency Meters (LR 47-59)

From time to time there have appeared well-written articles in the technical papers and trade journals describing applications of consistency meters and batchmeters. Both are of considerable interest to the technique in the production of concrete. However, we find many versions of the history of the origin of these meters.

Having had some part in the development of these devices, I have been asked to supply sufficient concise data for the record, so that laborious research will not be necessary to those that are interested in the subject.

The consistency meter, as a practical device to measure the consistency of concrete while still in the drum, was first mentioned in 1924 to Mr. Piepmeier, at that time assistant chief engineer of the Illinois State Highway Department, and later, chief engineer of the Highway Department of Missouri.

On February 26, 1925, at the annual meeting of the American Concrete Institute in Chicago, the idea and method was fully discussed with Frank H. Jackson, at that time Engineer of Tests, U. S. Bureau of Public Roads, Washington, D. C.; later the information was given to A. T. Goldbeck, Bureau of Public Roads. Thereafter, a series of experimental developments took place and a patent application for a consistency meter was filed on July 7, 1926. A basic patent covering any type of consistency indicator was issued Oct. 8, 1929, No. 1,730,893, to E. H. Lichtenberg, entitled

*A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 22, No. 1, Sept. 1950, *Proceedings*, V. 47. Separate prints of the entire Letters from Readers Section are available at 35 cents each. Address 18263 W. McNichols Rd., Detroit 19, Mich.

"Method of and Apparatus for Determining the Consistency of Concrete."

Since 1926 detailed developments of practical designs for different types of mixers by many men have evolved. O. G. Patch of the U. S. Bureau of Reclamation, formerly at Hoover Dam and later at Grand Coulee Dam, cooperating with W. H. Clagett, contributed much in perfecting a practical device for tilting type mixers. This particular device is the one generally used and represents more than 90 percent of all consistency meters in operation.

The batchmeter now generally used, a device which controls the mixing time in the drum and locks the discharge mechanism during mixing time, was developed in 1914 and 1915. An application for patent was filed Mar. 29, 1915, and basic patent No. 1,321,460 was issued Nov. 11, 1919, to E. H. Lichtenberg. Dozens of patents on detail designs have been issued since then. A great number of engineers and designers have lent their abilities to perfecting this device which, in turn, has had great influence in producing uniform concrete.

Both of the above described basic patents have long since expired. However, their usefulness is still going on and probably will continue for many years to come.

E. H. LICHTENBERG, Koehring Co., Milwaukee, Wis.

Relationship of Fly Ash and Corrosion (LR 47-60)

Industrial soot—fly ash—has recently found new economic uses which may make it a profitable by-product. Fly ash has been used in making concrete, bricks and in roadbed construction. But its use raises a question.

What are the possibilities of chemical and electrolytic corrosion of metallic members (beams, reinforcing, pipes) in concrete where fly ash has been used as an aggregate or a partial substitute for portland cement?

It appears that the use of materials which may have a high sulfur content in concrete may be highly objectionable from the standpoint of corrosive effect upon embedded metal.

READER

Proposed Refinement of Methods for Determining the Air Content of Concrete (LR 47-61)

Several methods of determining air content in concrete have been used, the "gravimetric" and the "pressure" methods being the best known and most widely accepted. These methods are dependent on either the assumption that accurate sampling can be accomplished, or on wet screening in an attempt to reduce sampling error.

From the available data on laboratory and field test results, it appears that the following conclusions are reasonable. Accurate sampling of concrete mixtures containing aggregate larger than 1½ in. is nearly impossible due to the segregation of the larger sizes in the mixer and during subsequent handling, particularly when a sample larger than that required for the actual volume of the testing device is obtained, which is usually the case. Example "A" illustrates the magnitude of this error.

Actual unit weight = 157.39 lb per cu ft.
Theoretical unit weight = 162.26 lb per cu ft.
Actual unit weight— + 1½ in. phase = 170.43 lb per cu ft.
Theoretical unit weight— − 1½ in. phase = 155.99 lb per cu ft.
Actual unit weight— − 1½ in. phase = 147.90 lb per cu ft.
Volume of + 1½ in. phase = 42.11 percent.
Volume of − 1½ in. phase = 57.89 percent.
Total volume of air = 3.001 percent.
Volume of air by volume of 1½ in. phase = 5.184 percent.

If a sampling error of 10 percent by volume of the + 1½ in. phase is made in the gravimetric method using a 3.5 cu ft measure:

3.5 × 42.11 percent × 0.90 = 1.327 cu ft + 1½ in. in measure.
3.5 − 1.327 = 2.173 cu ft − 1½ in. in measure.
1.327 × 170.43 = 226.16 lb + 1½ in. in measure.
2.173 × 147.90 = 321.39 lb − 1½ in. in measure.

EXAMPLE A

Given a concrete mix design, and data developed therefrom, as follows:

Materials	1 cu yd weight, lb	Unit weight, lb per cu ft
Cement	376.0	
Sand	613.8	166.06
No. 4-¾ in.	459.7	169.81
¾-1½ in.	703.9	177.30
1½-3 in.	769.0	170.43
3-6 in.	1168.8	170.43
Water	158.3	62.43
Air	3 percent by vol.	

226.16 + 321.39 = 547.55 lb total weight in measure.

547.55 ÷ 3.5 = 156.44 lb per cu ft in measure.

162.26 − 156.44 = 5.82 lb.

5.82 ÷ 162.26 = 3.61 percent air by volume of total mix.

3.61 ÷ 57.89 percent = 6.2 percent air by volume of the − 1½ in. phase.

6.2 − 5.2 = 1.0 percent error.

Obviously the results in the pressure method will be only as accurate as the sampling because the inclusion or exclusion of a few pieces of large aggregate in the relatively small devices used, such as the 2.8 cu ft size, will materially affect the proportions of the sample ingredients; and as the results of the pressure method depend on this relation, the answer will be correspondingly erroneous.

Wet screening is not looked upon with favor by many concrete technicians, as the screening of the wet concrete over a 1½ in. screen is considered likely to affect the quantity of air remaining in the 1½ in. phase. Apparently no adequate investigation of the effect of wet screening has been made; therefore, the magnitude of the induced change in air content is not definitely known. However, the results of such tests as have been made seem to indicate that the air change due to wet screening varies with changes in the mix proportions and type of admixture used.

In view of the importance of knowing the proportions of the sample as tested, it is proposed that the sample be wet screened *after* the air test has been made, either by the gravimetric or the pressure method, and that all the material in the testing device be screened over a 1½-in. screen and the + 1½ in. be washed over the 1½-in. screen in order to remove all adhering mortar and the weight of the + 1½ in. recorded. At this time the specific gravity of the + 1½ in. may also be determined if it is considered to be significantly variable, and the volume of the + 1½ in. may then be accurately computed. Then subtracting the weight and volume of the + 1½ in. from the weight and volume of the original sample, we have left the weight and volume of the − 1½ in., from which the unit weight of the − 1½ in. may then be determined. Subtracting this weight from the previously determined theoretical unit weight of the − 1½ in., and dividing by the theoretical unit weight will then give the percent air by volume of the − 1½ in. phase, thus eliminating any sampling error that might have been contributed by the + 1½ in. phase.

The following example will illustrate the computations necessary for the above test procedure, involving the same mixture proportions, basic data and sampling error as example "A".

EXAMPLE B

As calculated in example "A" involving a sampling error of 10 percent by volume of the plus 1½ in. phase:

547.55 lb = weight of sample in the 3.5 cu ft measure.

226.16 lb = weight of + 1½ in. in the 3.5 cu ft measure

547.55 − 226.16 = 321.39 lb − 1½ in. in the 3.5 cu ft measure.

321.39 ÷ 2.173 = 147.90 lb per cu ft − 1½ in. in the 3.5 cu ft measure.
155.99 − 147.90 = 8.09 lb.
8.09 ÷ 155.99 = 5.186 percent air by volume of − 1½ in.
5.186 × 57.89 percent = 3.0 percent by volume of the total mixture.

A. C. KAHL, U. S. Corps of Engineers, Pine Flat Dam, Fresno, Calif.

What Do We Know About Shotcrete? (LR 47-62)

There seems to be a dearth of accurate information about pneumatically-placed mortar. Here is a material that has been used for quite a long time, and which is being used a great deal today, and yet there is little information available as to permeability, bond, strength, shrinkage, etc. For example, the companies which advocate this material claim it has a high degree of impermeability and yet W. L. Chadwick* in the January 1947 JOURNAL states that it has greater permeability and relatively higher porosity than concrete. Could we not have more accurate data on this important engineering material?

R. A. SPENCER, Dean of Engineering, University of Saskatchewan, Saskatoon, Sask., Canada

Use Air-Entraining Cements Only in Exposed Parts of Structure (LR 47-63)

With the recognition of the advantages in the use of air-entraining cements and air-entraining compounds as admixtures applied at the mixer, resulting in much greater durability, this procedure has gained the tremendous popularity it deserves. Like all other blessings, this one is not an entirely unmixed blessing for air entrainment is accompanied with a loss of compressive strength nearly in direct proportion with the volume of air entrained in the concrete. This is one of the reasons that specifications contain a provision for limiting the amount of air entrained in the concrete from 4 to 6 percent of the volume of the concrete, so that the loss of compressive strength may be limited to about 10 percent (maximum) of the original design strength. For that reason, the cement content for a desired strength concrete must be increased when air entrainment is required.

However, durability is a factor that only enters into consideration for concrete that is exposed to the elements (or chemical fumes, etc., in interior locations), and it should certainly be feasible to plan the concrete work on the project so that air-entraining cements or admixtures are used only on that part of the project which is subject to exposure from the elements, and use the "normal" type for the remainder.

Under certain conditions, both types of cement or concrete with and without admixture could be used on the same member of a structure. The writer has in mind the heavy concrete slabs used in railroad construction. It is not at all unusual to find a slab 24 to 36 in. thick on railroad undercrossings; the only portion that is subject to attack by the elements is the underside and the sides, the top being generally protected by waterproofing membrane and a protection coat, and in addition completely covered by ballast or fill. The concrete on the underside is not subject to compression so that the loss of strength there would not enter into consideration, while the elimination of the air-entraining admixture in the rest of the structure would mean retaining the design strength of the concrete without the necessity of increasing the cement content.

This procedure would result in a saving of cement, eliminate the necessity of revising the design proportions, but would require rigid inspection of each batch of concrete mixed in the course of the "pour" of the slab, which latter would be a blessing because as a rule we get too little inspection.

M. HIRSCHTHAL, Consulting Engineer, New York, N. Y.

*Chadwick, W. L., "Hydraulic Structure Maintenance Using Pneumatically Placed Mortar," ACI JOURNAL, Jan. 1947, *Proc.* V. 43, pp. 533-548.

Design of an Intake Structure for an Arctic Water Supply (LR 47-64)

The writer is currently faced with the problem of designing an intake structure to be used in an arctic water supply system. The inside dimension of the cylindrical shaft is to be 21 ft. It will have a maximum head of 45 ft above the bedrock foundation (which is exposed at this site), and will be subjected to loads of 5 ft thick sheets of ice throughout most of its height.

It will serve an impounding reservoir that will be fed by a large creek throughout the summer months and will be gradually drained over a period of seven months when the creek will be frozen solid. Consequently, the ice sheet will act, throughout a good portion of its height, against the tower—perhaps unevenly. The mean minimum temperature of the area is 15 F; the mean maximum temperature, 27 F; the lowest recorded temperature is −41 F; and the highest recorded temperature was 75 F.

Inasmuch as this problem has unique design conditions, the writer wonders if any JOURNAL readers could advise the proper method of approach gained from similar cases.

READER

Prestressed Concrete Sleepers (Railroad Ties) (LR 47-65)

The shortage of timber railway sleepers is practically world-wide, particularly in Great Britain. Official estimates state that Britain alone needs more than one million sleepers per year, with a deficit of many thousands, and other countries are in a similar position although to a lesser degree.

Steel sleepers are almost unobtainable also. The only suitable substitute available (particularly in tropical countries troubled with termites) is concrete sleepers, which are easily and rapidly produced. Much literature has appeared on concrete sleepers and experience has shown that an ordinary concrete sleeper with mild steel reinforcement fails under traffic other than the slowest moving (*i.e.*, for light siding work). The writer recently published an article in the *Royal Engineers Journal* (London). V, 63, June 1949, describing how these difficulties have been overcome with prestressing of the concrete.

To all intents sleepers can be divided into two classes: (1) positive anchored and (2) nonpositive.

It is not the intention of this article to discuss the merits of the two different systems although the writer prefers the positive-anchored type for the following reasons: (1) Positive-anchored type sleepers denote a positive anchorage to maintain the steel in tension, whereas the nonpositive type is dependent on maintaining tension by adhesion of the steel and concrete through bond only. (2) High tensile steel drawn wire, which comprises the reinforcement, possesses a highly smooth surface which is difficult to indent to improve bond. (3) The steel and concrete bond may be affected during the life of the sleeper by many circumstances such as fatigue in the concrete or steel from constant "hammering" with continuous loads, or by the varying forces in opposition, namely contraction and expansion of concrete and steel, particularly where there is a rapid succession of temperature changes.

Prestressed concrete sleepers are now being accepted by the British railroads for 70-80 mph traffic and it can therefore be assumed that this type of sleeper is here to stay. A British Standard Specification (War Emergency 986, 1945) covers prestressed concrete sleepers where the anchorage of the steel is by bond alone, but revisions are being prepared to cover the positive-anchored type.

It is intended, when the steel situation is less acute, that the existing design of prestressed sleepers will be modified so that fewer high tensile steel wires of larger diameter will be substituted for the 16 wires now used. The ultimate tensile stress of the wire is 100-110 tons per sq in. and the tension on the wire is applied through straining devices to a load of 40 tons, or 2½ tons per wire.

The normal concrete mix is 1:1½:3 with a slump of ¼ in. which gives more than 4000 psi concrete in 24 hours. This rapid development of strength is due to properly controlled steam curing.

The operation of a plant that is capable of producing 200 sleepers per day is described below.

The wire is bent to jigs and looped around the cast iron anchor blocks; the two "bobbed" ends of the wire lie side by side in the groove of the anchor block. On these ends a mild steel plate is placed and this, in turn, is surmounted by a standard square nut which is screwed to the end of a tensioning bolt. Tension is applied through the bolt and thus the wires are compressed between the mild steel plate and the groove in the cast iron block. The wedge action securely locks the wire and no slip can occur; obviously the greater the pull on the tension bolt, the greater the wedge action.

To facilitate production, an assembly line is provided and the molds move along the line as the various operations are performed. The molds are filled from a specially designed mixer and are vibrated during this operation to ensure perfect consolidation. A stiff concrete mix in conjunction with the special mixing arrangements, vibrating and steam curing make possible a compressive strength of 4000 psi in less than 24 hours.

After completing the actual assembly and filling operation, the molds filled with concrete enter a steam curing chamber with a temperature of 160 F. Automatic recording devices outside the steam chamber ensure an efficient control of temperature and humidity during this operation.

Test cubes are made under identical conditions as the sleepers and after these cubes test 4000 psi the tension bolts are removed from the sleepers as the concrete is then capable, with the positive anchorage, of maintaining the 40 tons tension. The sleepers are removed from the molds and the molds returned to the assembly line.

Sleepers are often in trucks ready for shipment within 24 hours of being cast. The sleepers are tested before shipment in accordance with British Standard Specifications.

W. B. Sykes, Stent Precast Concrete Ltd., Dagenham Dock, Essex, England

Paneled Ceiling Loaded by Concentrated Loads at the Intersections of the Beams (LR 46-37)*

In the December 1949 Journal L. S. Muller presented a particular solution to a problem which deserves to be treated in a more general manner and the solution put into a shape to allow the application in each case of loading.

Taking into consideration that not all crossing points may be subjected to concentrated loads there will be two cases:

Case 1—The crossing point is subjected to a concentrated load P_{kz}. The distributed loads are:

$\alpha_{kz} P_{kz}$ for a beam of span L_x

$(1 - \alpha_{kz}) P_{kz}$ for a beam of span L_y

Case 2—The crossing point is not loaded. The distributed loads are:

α_{kz} for a beam of span L_x and

$-\alpha_{kz}$ for a beam of span L_y

as it is sufficient to distribute a load of unit value.

Eq. (3) of Mr. Muller's solution will be

$$f_{ki} = \frac{l_x^3}{6\,E\,I\,(m+1)} \left(\sum_{z=1}^{i} (\alpha P)_{kz} z\,(m+1-i) \right.$$

$$\left. [2\,(m+1)\,i - i^2 - z^2] + \right.$$

$$\left. + \sum_{z=i+1}^{m} (\alpha P)_{kz}\, i\,(m+1-z) \right.$$

$$\left. [2\,(m+1)\,z - i^2 - z^2] \right.$$

or in a more convenient form

$$f_{ki} = \frac{l_x^3}{6\,E\,I} \sum_{z=1}^{m} (\alpha P)_{kz}\, X_{ki,\,kz}\dagger \ldots\ldots\ldots\ldots (1)$$

where

$$X_{ki,\,kz} = \frac{z\,(m+1-i)\,[2\,(m+1)\,x - i^2 - z^2]}{m+1}$$

when $z \leqq i$

or

$$X_{ki,\,kz} = \frac{i\,(m+1-z)\,[2\,(m+1)\,i - i^2 - z^2]}{m+1}$$

when $z \geqq i$

Mr. Muller's Eq. (4) is superfluous as it is the same as his Eq. (3), being applied to each row of beams of span L_x and L_y respectively.

In Eq. (1), $(\alpha P)_{kz}$ denotes in Case 1:

$\alpha_{kz} P_{kz}$ for a beam of span L_x

*See also ACI Journal, Dec. 1949, *Proc.* V. 46, p. 293 and Mar. 1950, *Proc.* V. 46, p. 557.

†(ki), (kz) are coordinates of the points in question.

or $(1 - \alpha_{kz})\ P_{kz}$ for a beam of span L_y and in Case 2:

α_{kz} for a beam of span L_z

or $-\alpha_{kz}$ for a beam of span L_y

The coefficients $X_{ki,\ kz}$, as it may be seen from Eq. (1), are *proportional to the influences* on the deflection of the point ki, when a concentrated load of unit value is placed at the point kz, considered as a point of the beam k with a span L_z.

Eq. (1) can also be written:

$$f_{ki} = f_{ki,\ k1} + f_{ki,\ k2} + f_{ki,\ k3} + \ldots + f_{ki,\ km}$$

$$= \sum_{z=1}^{m} f_{ki,\ kz}^{*}$$

i.e., as a *sum of the partial deflections*, each partial deflection being

$$f_{ki,\ kz} = \frac{l_x^3}{6\,E\,I}\,(\alpha P)_{kz}\,X_{ki,\ kz}$$

The coefficients $X_{ki,\ kz}$ comply with Maxwell's theorem

$$X_{ki,\ kz} = X_{kz,\ ki}$$

and being dependent only on m they may be computed once.

Below the values are given for $X_{ki,\ kz}$ for successive values of m from $m = 1$ to $m = 7$.† Fig. 1 shows the arrangement for $m = 1$. There is: $k = k$ and $i = z = 1$, therefore $X_{ki,\ kz} = 1$. The single equation will be

$$f_{11} = \frac{l_x^3}{6\,E\,I}\,(\alpha P)_{11}$$

The arrangement in Fig. 2 is for $m = 2$.

$k = k$ $i = 1$ $z = 1$ $X_{k1,\ k1} = \frac{1}{3} \times 8$

$i = 1$ $z = 2$ $\ldots . X_{k1,\ k2} = \frac{1}{3} \times 7$

$i = 2$ $z = 1$ $X_{k2,\ k1} = \frac{1}{3} \times 7$

$i = 2$ $z = 2$ $\ldots . X_{k2,\ k2} = \frac{1}{3} \times 8$

The two equations are

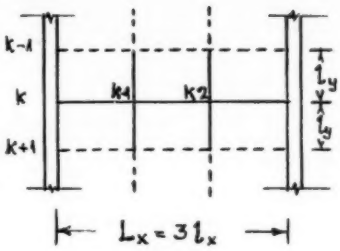


Fig. 2

$$f_{k1} = \frac{l_x^3}{6\,E\,I} \times \tfrac{1}{3}\,[8\,(\alpha P)_{k1} + 7\,(\alpha P)_{k2}]$$

$$f_{k2} = \frac{l_x^3}{6\,E\,I} \times \tfrac{1}{3}\,[7\,(\alpha P)_{k1} + 8\,(\alpha P)_{k2}]$$

The coefficients $X_{ki,\ kz}$ may be written as a matrix.

$$\tfrac{1}{3} \left\| \begin{matrix} 8 & 7 \\ 7 & 8 \end{matrix} \right\|$$

The arrangement for $m = 3$ is shown in Fig. 3. The matrix of coefficients $X_{ki,\ kz}$ is now:

$$\tfrac{1}{4} \left\| \begin{matrix} 18 & 22 & 14 \\ 22 & 32 & 22 \\ 14 & 22 & 18 \end{matrix} \right\|$$

The matrix of the coefficients $X_{ki,\ kz}$ is for:

$m = 4$

$$\tfrac{1}{5} \left\| \begin{matrix} 32 & 45 & 40 & 23 \\ 45 & 72 & 68 & 40 \\ 40 & 68 & 72 & 45 \\ 23 & 40 & 45 & 32 \end{matrix} \right\|$$

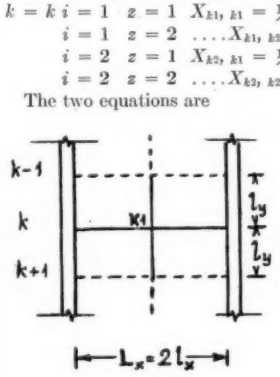


Fig. 1

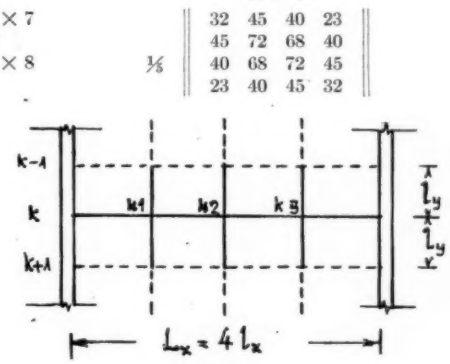


Fig. 3

*To be read—deflection of point ki caused by a load placed at point kz.

†The highest value is set up arbitrarily.

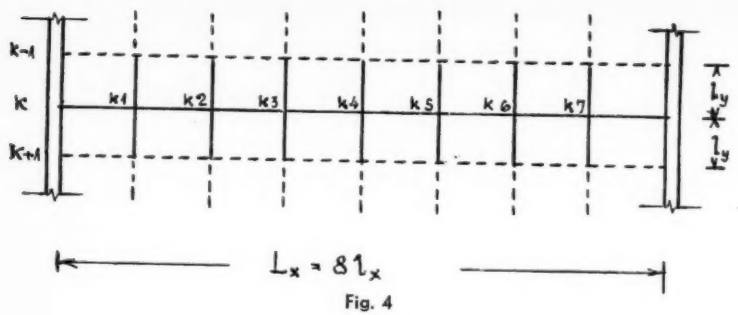


Fig. 4

$m = 5$

$$\frac{1}{6}\begin{Vmatrix} 50 & 76 & 78 & 62 & 34 \\ 76 & 128 & 138 & 112 & 62 \\ 78 & 138 & 162 & 138 & 78 \\ 62 & 112 & 138 & 128 & 76 \\ 34 & 62 & 78 & 76 & 50 \end{Vmatrix}$$

$m = 6$

$$\frac{1}{5}\begin{Vmatrix} 72 & 115 & 128 & 117 & 88 & 47 \\ 115 & 200 & 232 & 216 & 164 & 88 \\ 128 & 232 & 288 & 279 & 216 & 117 \\ 117 & 216 & 279 & 288 & 232 & 128 \\ 88 & 164 & 216 & 232 & 200 & 115 \\ 47 & 88 & 117 & 128 & 115 & 72 \end{Vmatrix}$$

$m = 7$

$$\frac{1}{8}\begin{Vmatrix} 98 & 162 & 190 & 188 & 162 & 118 & 62 \\ 162 & 288 & 350 & 352 & 306 & 224 & 118 \\ 190 & 350 & 450 & 468 & 414 & 306 & 162 \\ 188 & 352 & 468 & 512 & 468 & 352 & 188 \\ 162 & 306 & 414 & 468 & 450 & 350 & 190 \\ 118 & 224 & 306 & 352 & 350 & 288 & 162 \\ 62 & 118 & 162 & 188 & 190 & 162 & 98 \end{Vmatrix}$$

Note: $X_{k1,\,k7} + X_{k1,\,k(s+2)} = X_{k2,\,k(s+1)}$

$98 + 190 = 288$

$188 + 118 = 306$

Arrangements for values $m = 4$ to $m = 7$ are shown in Fig. 4.

Example

Considering the same example as on p. 297 of the December 1949 JOURNAL (Fig. 3 of Mr. Muller's letter) we may write the equations required in a symbolic form ($m = 3$, $n = 4$).

$$\frac{l_x^3}{6EI} \times \frac{1}{4}\begin{Vmatrix} 18 & 22 & 14 \\ 22 & 32 & 22 \\ 14 & 22 & 18 \end{Vmatrix} =$$

$$\frac{l_y^3}{6EI} \times \frac{1}{5}\begin{Vmatrix} 32 & 45 & 40 & 23 \\ 45 & 72 & 68 & 40 \\ 40 & 68 & 72 & 45 \\ 23 & 40 & 45 & 32 \end{Vmatrix}$$

Omitting l_x, l_y, $6EI$ and assuming that *only* the crossing point, (2, 3) be loaded we get the following 12 equations:

$$\begin{aligned}
&\tfrac{1}{4}\,(18\alpha_{11} + 22\alpha_{12} + 14\alpha_{13}) = \\
&\quad \tfrac{1}{5}\,(-32\alpha_{11} - 45\alpha_{21} - 40\alpha_{31} - 23\alpha_{41}) \\
&\tfrac{1}{4}\,(22\alpha_{11} + 32\alpha_{12} + 22\alpha_{13}) = \\
&\quad \tfrac{1}{5}\,(-32\alpha_{12} - 45\alpha_{22} - 40\alpha_{32} - 23\alpha_{42}) \\
&\tfrac{1}{4}\,(14\alpha_{11} + 22\alpha_{12} + 18\alpha_{13}) = \\
&\tfrac{1}{5}\,[-32\alpha_{13} + 45\,(1 - \alpha_{23})\,P - 40\alpha_{33} - 23\alpha_{43}]
\end{aligned}$$

$$\begin{aligned}
&\tfrac{1}{4}\,(18\alpha_{21} + 22\alpha_{22} + 14\alpha_{23}\,P) = \\
&\quad \tfrac{1}{5}\,(-45\alpha_{11} - 72\alpha_{21} - 68\alpha_{31} - 40\alpha_{41}) \\
&\tfrac{1}{4}\,(22\alpha_{21} + 32\alpha_{22} + 22\alpha_{23}\,P) = \\
&\quad \tfrac{1}{5}\,(-45\alpha_{12} - 72\alpha_{22} - 68\alpha_{32} - 40\alpha_{42}) \\
&\tfrac{1}{4}\,(14\alpha_{21} + 22\alpha_{22} + 18\alpha_{23}\,P) = \\
&\tfrac{1}{5}\,[-45\alpha_{13} + 72\,(1 - \alpha_{23})\,P - 68\alpha_{33} - 40\alpha_{43}]
\end{aligned}$$

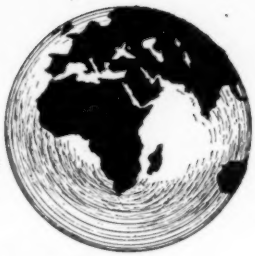
$$\begin{aligned}
&\tfrac{1}{4}\,(18\alpha_{31} + 22\alpha_{32} + 14\alpha_{33}) = \\
&\quad \tfrac{1}{5}\,(-40\alpha_{11} - 68\alpha_{21} - 72\alpha_{31} - 45\alpha_{41}) \\
&\tfrac{1}{4}\,(22\alpha_{31} + 32\alpha_{32} + 22\alpha_{33}) = \\
&\quad \tfrac{1}{5}\,(-40\alpha_{12} - 68\alpha_{22} - 72\alpha_{32} - 45\alpha_{42}) \\
&\tfrac{1}{4}\,(14\alpha_{31} + 22\alpha_{32} + 18\alpha_{33}) = \\
&\tfrac{1}{5}\,[-40\alpha_{13} + 68\,(1 - \alpha_{23})\,P - 72\alpha_{33} - 45\alpha_{43}]
\end{aligned}$$

$$\begin{aligned}
&\tfrac{1}{4}\,(18\alpha_{41} + 22\alpha_{42} + 14\alpha_{43}) = \\
&\quad \tfrac{1}{5}\,(-23\alpha_{11} - 40\alpha_{21} - 45\alpha_{31} - 32\alpha_{41}) \\
&\tfrac{1}{4}\,(22\alpha_{41} + 32\alpha_{42} + 22\alpha_{43}) = \\
&\quad \tfrac{1}{5}\,(-23\alpha_{12} - 40\alpha_{22} - 45\alpha_{32} - 32\alpha_{42}) \\
&\tfrac{1}{4}\,(14\alpha_{41} + 22\alpha_{42} + 18\alpha_{43}) = \\
&\tfrac{1}{5}\,[-23\alpha_{13} + 40\,(1 - \alpha_{23})\,P - 45\alpha_{33} - 32\alpha_{43}]
\end{aligned}$$

With the aid of mechanical equipment it is easy to solve these 12 equations and to distribute the single load over the whole grid of beams.

ISSER HARBAND, C.E.,
Jerusalem, Israel

Current Reviews*



of Significant Contributions in Foreign and Domestic Publications

Analysis of Vierendeel trusses (Contributo al calcolo delle travi Vierendeel a correnti paralleli uguali)

EMILIO PASCALE, *Giornale del Genio Civile* (Rome), V. 87, No. 8 and 9, July-Aug. 1949, pp. 376-384
Reviewed by GENNARO MIANULLI

This is a new approach to the design of Vierendeel trusses but limited only to those trusses with parallel chords and of equal lengths. Tables are included for practical application and the method is illustrated with a few examples.

Influence lines for continuous beams (Equagioni e proprieta' analitiche delle linee d'influenza nella frave continua)

ENRICO CASTIGLIA, *Giornale del Genio Civile* (Rome), V. 87, No. 9, Sept. 1949, pp. 423-441
Reviewed by GENNARO MIANULLI

A solution of influence lines equations for moment and shear in any section of a continuous beam on unyielding supports is presented in this article. The influence lines are obtained by simple derivations. Tables are included for continuous beams up to five spans.

Hoisting, transport and support of reinforced concrete piles (Oplegging, transport en hijsen van heipalen van gewapend beton)

E. A. F. HUBER, *Cement*, No. 15-16, 1950, pp. 320-327
Reviewed by J. W. T. VAN ERP

Piles of great length require special care in handling in regard to their ability to resist bending. The author reviews various ways of supporting them at 2, 3 or more points as well as the proper way of hoisting them. A comparison between normally reinforced and prestressed piles under these conditions of loading is made.

Quenching of clinkers from rotary kilns

R. G. UHLIG, *Zement-Kalk-Gips*, V. 3, No. 2, Feb. 1950, pp. 30-35
Reviewed by WERNER H. GUMPERTZ

This article discusses the economic and technical advantages of clinker quenching, as done with equipment of the Fuller Company in Catasaqua, Pa., and in Germany. It is in a reply to an article by W. Anselm, in *Zement-Kalk-Gips*, V. 2, May 1949, p. 94. The controversial point is the most economical use of the heat removed by the cooling air.

The effect of retarding agents on redundant prismatic solids (Sulla valutazione degli effetti dell'elasticita' ritardata nei solidi iperstatici)

FRANCESCO LEVI, *Giornale del Genio Civile* (Rome), V. 87, No. 7-8, July-Aug. 1949, pp. 343-350
Reviewed by GENNARO MIANULLI

The author proposes an interpretation of the statical behavior of elasticity when retarded by creep, shrinkage and plastic flow. A new stress-deformation law is deduced and applied to the behavior of materials under load.

The treatment is theoretical, but the author has succeeded in simplifying formulas. This is a step further in the attempt to investigate what stresses are set up in concrete and steel when shrinkage or volume changes take place in a structural member.

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Weather and the building industry

Research Conference Report No. 1, Building Research Advisory Board, National Research Council, Washington 25, D. C., 160 pp., $3.50

The present knowledge of climatology and its relation to the building industry is summarized and the present status of research in building material and equipment is analyzed. Included are reports on recent climatological data and discussions of how the weather sciences can be developed for better use of the building industry. Climate is discussed in relation to structures, design of buildings and indoor comfort.

Harbor reconstruction for the port of the Canal of Terracina (Ricostruzione della Darsena e del Canale del Porto di Terracina)

GIUSEPPE STRONGOLI, *Giornale del Genio Civile* (Rome), V. 87, No. 6, June 1949, pp. 316-323

Reviewed by GENNARO MIANULLI

This article is of interest to engineers engaged to design ports for small communities.

The author gives a brief history, a description of the location and the reasons for choosing the type of structures for the conditions encountered. The interesting features are complete soil data, design of concrete gravity quay walls or bulkheads and sheet piling for shore protection of the canal.

Rigid frame design (Il calcolo dei portali costituiti da membratura molto flessibili)

ALBERTO GROSSI, *Giornale del Genio Civile* (Rome), V. 87, No. 8 and 9, July-Aug. 1949, pp. 351-360, No. 11, Nov. 1949, pp. 583-594

Reviewed by GENNARO MIANULLI

The author discusses the general and special mathematical methods and analytical procedure for determining the values of redundant elements in rigid frames and its limitations. By means of models a frame with semi-rigid members was investigated under gravity load only and checked analytically. Tables and diagrams have been compiled for this type of frame and made part of the test.

Hardy Cross moment distribution method (Di una generalizzazione del metodo di Cross)

BRUNO DALL'AGLIO, *Giornale del Genio Civile* (Rome), V. 87, No. 6, June 1949, p. 309, No. 8 and 9, July-Aug. 1949, p. 372

Reviewed by GENNARO MIANULLI

The author combines and simplifies Cross' and Grinter's methods by assuming that the beam displacement will not take place in determined phases of the procedure, but will always be allowed. The beam displacement and joint rotations are evaluated simultaneously. A new method is derived which can be applied in analyzing any type of frame. Two investigations of a multistory and multispan frames illustrate the procedure proposed.

The rectangular foundation slab (Die rechteckige Gruendungsplatte)

F. HALBRITTER, *Die Bautechnik* (Berlin), V. 26, No. 6, June 1949, pp. 181-186

Reviewed by RUDOLPH FISCHL

Based on the design method for flat slabs developed by Grein, the computation of an infinitely long foundation strip with uniformly distributed soil pressure is presented in this paper.

The stresses for a single rectangular slab are developed from the general case by taking cuts and superimposing the bending moments at these sections with negative signs. An approximation is given beside the exact method. Eight figures and eight tables are included.

Bibliographic review of agents and causes of concrete corrosion (Apercu bibliographique sur les agents et les causes de la corrosion du beton)

A. NICOL, *Revue des Materiaux*, No. 414, 1950, pp. 65-84 and No. 415, 1950, pp. 111-126

Reviewed by PHILLIP L. MELVILLE

A very complete review of the agents and causes of concrete disintegration. Includes the effects of freezing; fire; heat; sea, subterranean, marsh and spring waters; gases (especially in connection with sewage); organic compounds (such as oils, glycerin, sugar, molasses, beer, ink and wine). Means of preventing disintegration are expounded. Use of sound mixing waters, air-entraining agents, waterproofing compounds, special cements, is discussed. The article ends with a series of 128 references.

Studies of highway skew slab-bridges with curbs—Part II: Laboratory research

M. L. GOSSARD, C. P. SIESS, N. M. NEWMARK, and L. E. GOODMAN, *University of Illinois Engineering Experiment Station Bulletin* 386, Feb. 1950

Reviewed by C. P. SIESS

The results of laboratory tests on four skew slab-bridges of reinforced concrete are reported. Three of the models had 45-degree angles of skew, and were half-scale, with right span lengths of 9 ft 9 in. The

fourth model had a 60-degree angle of skew, and was fifth-scale, with a right span length of 5 ft $10\frac{5}{8}$ in. A principal variable in the test was the amount and arrangement of reinforcement in the various bridges. All bridges were tested at working loads and to failure under loadings corresponding to the H-20 truck of the AASHO.

The ultimate span of arches (Die groesstmoeglichen Spannweiten von Bogentraegern)

G. Brendel, *Beton- u. Stahlbetonbau* (Berlin), V. 45, No. 1, Jan. 1950, pp. 10-16
Reviewed by Rudolph Fischl

The theoretical ultimate span for arches is given by the simple relation $l_{max} = \lambda K$, where λ is dependent upon the geometric shape, and K is the ratio of stress and weight of the material of the arch. The presentation of the factor λ with all its variations and special cases is given in this article.

The practical ultimate span is far beyond the theoretical value. Its determination is difficult and, considering the various influences, highly unsafe. Curves are given as help for estimating the factor V in the relation $L_{prac} = VL_{theor}$. The paper is illustrated by eleven graphs and three tables.

Soil drainage by porous concrete pipe (Geen drainage problemen met poreuze betonbuizen)

G. J. Hamer, *Cement*, No. 15-16, 1950, pp. 338-346
Reviewed by J. W. T. Van Erp

In the Netherlands a large portion of its arable land lies below sea level. This necessitated artificial drainage on a large scale and led to the development of its unique system of polders. These polders exist and maintain their intensive agrarian production only because of a continually alert de-watering system. For instance, in the newly reclaimed Zyuderzee polders porous concrete drain pipe will be part of that drainage system. These pipes, made according to a patented process will be extensively used. When installed they present an entirely flush in- and outside surface, being jointed by a gasket of pressed peat. This gasket swells up as soon as the drains are laid, thereby forming a tight joint that prevents the silting up of the pipe. Several further advantages are claimed over the old tile drains, *i.e.*, greater strength, uniformity and durability.

Hyperbolic-paraboloid concrete roofs in Czechoslovakia

K. Hruban, *Concrete and Constructional Engineering*, (London), V. 44, No. 8, Aug. 1949, pp. 247-249
Reviewed by Ivan M. Viest

Thin-shell construction of reinforced concrete roofs has been used extensively in Czechoslovakia in the past few years. The author describes two types of large factory roofs: cast-in-place and precast structures.

The cast-in-place type consists of a number of $2\frac{3}{4}$-in. thick reinforced concrete shells shaped as a hyperbolic paraboloid (shape of a saddle). The shells are supported by dividing parabolic tied arches resting on columns. The precast roofs are of the saw-tooth type. Each element consists of a hyperbolic-paraboloid shell $1\frac{3}{8}$ in. thick surrounded by a frame of reinforced concrete ribs. The weight of an element is 6 tons.

Influence charts for concrete pavements

Gerald Pickett and G. K. Ray, *Proceedings*, A.S.C.E., V. 76, Separate No. 12, Apr. 1950, 25 pp., $0.50

Influence charts are presented for the purpose of obtaining theoretical deflections and moments of pavement slabs under load. The charts are similar to, and are used in the same manner as, those presented by Nathan M. Newmark, for the determination of theoretical stresses and displacements in elastic foundations.

Points at a slab edge, points near a slab edge, and points far from any slab edges are considered. Both the assumption of a subgrade modulus as originally proposed by H. M. Westergaard, and the assumption of a deep elastic solid considered by Mr. Westergaard and others are treated. The charts are based on the theoretical equations derived by Mr. Westergaard and by A. H. A. Hogg.

Test to determine the interaction of prefabricated elements of reinforced concrete floors.

Herman Amos and Walter Bochmann, *Deutsches Ausschuss Stahlbeton*, No. 101, 1948, pp. 59-84
Applied Mechanics Reviews Apr. 1950 (Reiner)

Prefabricated reinforced-concrete floors, consisting of reinforced-concrete beams 75 cm center to center and nonreinforced slabs 32 cm wide resting on the beams, were tested for ultimate load when the joints between beams and slabs were either dry or connected by mortar, and slabs were connected to the beams by steel rods 30 cm apart or not so

connected. It was found that the provision of such steel cross connection increases the ultimate load by 40 percent while mortaring-in of the joints gives an increase of 7 percent. German regulations therefore require for prefabricated slabs the provision of such connecting rods at right angles to the beams.

Analysis of truncated elliptical shells (Calcul des coupoles elliptiques surbaissees)

R. Chambaud, *Annales de L'Institut Technique du Batiment et des Travaux Publics*, No. 133, New Series, *Manuel du Beton Arme*, No. 19, May 1950
Reviewed by C. P. Siess

An analysis of a roof structure consisting of a truncated ellipsoidal shell or dome is presented. The shell is supported at its edges by an elliptical ring girder assumed to be flexible. The girder acts in tension only to resist the horizontal thrusts from the shell; only vertical loads are transmitted to the supports. The shell is considered to be subject only to forces within its plane; it is assumed to be flexible so that bending stresses may be neglected. The analysis is approximate but both simple and rational. Elastic deflections of the shell are discussed, as well as deformations due to creep and shrinkage of the concrete. Buckling is considered as imposing a limitation in the thickness of the shell. A numerical example is included.

Thermal and moisture expansion studies of some domestic granites

Arthur Hockman and Daniel W. Kessler, *Journal of Research*, V. 44, Apr. 1950
Reviewed by C. C. Fishburn

As a part of a study of the physical properties of building stone, thermal expansion determinations were made on 48 samples of domestic granites by the differential interferometer method (with an interferograph recording attachment) over the temperature range -20 to $+60$ C. Thermal coefficients computed between -20 and $+60$ C ranged from 4.8 to 8.3 $\times 10^{-6}$ per deg C with an average of 6.2 $\times 10^{-6}$ per deg C. Coefficients obtained on cooling (60 to 0 C) averaged 6.7 $\times 10^{-6}$ per deg C. Expansion curves drawn for all samples indicate slight irregularities in the 0- to 20-degree range in the heating curves of at least 65 percent of the samples. These irregularities are probably caused by moisture changes in the sample during the test.

Moisture expansion determinations were made on the 48 samples by means of the Tuckerman optical strain gage over a 24-hour period at constant temperature. The expansions obtained ranged from 0.0004 to 0.009 percent and averaged 0.0039 percent.

Adverse effects resulting from continued thermal and moisture expansion and contraction of granite are believed to be of some significance in monumental structures that are expected to last for long periods of time.

Influences on concrete

A. Kleinlogel (Translated from the 1941 Edition by F. S. Morgenroth), Frederick Ungar Publishing Co., New York., 281 pp., $7.50

A wide range of practices and substances affecting the stability of concrete are considered in alphabetical order preceded by brief basic rules for avoiding damage to concrete and reinforced concrete structures. In addition to the basic ingredients of concrete the effects of many chemical substances in contact with concrete structures are given as well as corrective measures to prevent deterioration of the concrete.

Despite the fact that most of the references antedate 1939 and that proprietary compounds and concrete practices mentioned are principally of German origin the book is an interesting source of brief information on the effects of many substances on and in concrete about which relatively little data are readily available elsewhere.

Investigation of the stability of the trisulfate form of the calcium sulfoaluminate (Recherches sur la stabilite du sel de Candlot)

H. Lafuma and J. Brocard, *Technical Publication* No. 15, Centre d'Etudes et de Recherches de l'Industries des Liants Hydrauliques (Paris), 1950
Reviewed by George L. Kalousek

The authors observe that $3CaO \cdot Al_2O_3 \cdot 3C\bar{A}SO_4 \cdot 31H_2O$ is stable at room temperatures, transforms to $3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$ at approximately 60 to 70 C and to the cubic $3CaO \cdot Al_2O_3 \cdot 6H_2O$ and gypsum at 150 C in an autoclave.

Initial and final setting of slag and portland cement is accelerated over the entire temperature range of about 20 to 100 C; high alumina cement sets most slowly at 30 C and almost instantaneously at 50 C; sursulfate cement failed to develop final set

between 80 to 100 C; otherwise being similar to slag cement. The anamolous setting and low strength of sursulfate cement, swelling of mortars made of H.R.I. portland, slag and sursulfate cement upon heating may be attributed to the described reactions of the calcium sulfoaluminates.

Watertightness of structures—I (L'Etancheite des ouvrages)

M. Duriez, *La Technique Moderne-Construction* (Paris), V. 5, No. 2, Feb. 1950, pp. 40-45
Reviewed by Alexander M. Turitzin

The question of the mechanism of watertightness of structural materials is treated in a manner which should be of interest to the physicist as well as to the cement technician. The major topics are observations under an electronic microscope of the composition, stability and evolution of both ionic binding agents such as cements, clays, and of nonionic binding agents such as tars, bitumens and organic plastic materials; penetration and retention of water in construction materials by absorption and capillarity; the relation of the fineness of the cement and its pH to the watertightness of the concrete surfaces; the phenomena of corrosion and their relation to the watertightness of structures; the future of rubber and plastic materials as agents in the solution of problems involving watertightness; the relation between the hardness and brittleness of aggregate and asphaltic mixes. A glossary of scientific terms is appended.

Watertightness of structures—II (L'Etancheite des ouvrages)

M. Duriez, *La Technique Moderne-Construction* (Paris) V. 5, No. 3, Mar. 1950, pp. 83-86
Reviewed by Alexander M. Turitzin

The author discusses the relation of the watertightness to the heat insulation of flat roofs. To make living in the apartment just below the roof as comfortable as in an apartment located in the lower stories, three conditions must be satisfied, *viz*: first, water condensation under the flat roof must be avoided; second, loss of heat during the winter days must be eliminated and therefore a saving in fuel realized; and third, the effect of the radiant heat of the sun must be reduced during the summer season. Formulas for heat transfer are derived and the effect of the velocity of the wind, the humidity of the air, the surface of absorption of the sun's rays, the conductivity of the insulation, etc., are analyzed.

To extend the life of the waterproofing membrane on a flat roof, the author proves that a layer of concrete between the waterproofing membrane and the heat insulating material is imperative. A numerical example, showing the saving in the weight of coal produced by a properly waterproofed and insulated flat roof, is given.

Temperature stresses in concrete pavements

Sven G. Bergstrom, *Proceedings* No. 14, Swedish Cement and Concrete Research Institute (Stockholm), 37 pp.

This paper presents methods, formulas and diagrams for the calculation of temperature stresses in concrete pavements. Special attention has been given to the tensile stresses at the bottom of the pavement caused by temperature variations. These stresses are of particular interest in the design of nonreinforced and crack-reinforced pavements, and the results can be used for determining the distance between the warping joints and the contraction joints. Furthermore, the paper deals with the tensile stresses at the top of the pavement, which are of importance in the design of statically reinforced pavements. Finally, an account is given of a theory which can be utilized for determining the maximum permissible distance between the expansion joints, so as to prevent the compressive stresses from causing separation of the pavement from the subgrade and blow-up.

Power savings and production increase through concentric compartmentation of rotary grinding mills for cement

E. Pralle, *Zement-Kalk-Gips*, V. 3, No. 2, Feb. 1950, pp. 35-36
Reviewed by Werner H. Gumpertz

Concentric compartmentation is usually built into the second and third chambers of a rotary grinding mill. Subdivision into about five sections with approximate balancing of the weight of the grinding balls reduces the weight to be lifted and at the same time doubles the grinding area in the compartment. Total power requirements are given as 90 kwh per metric ton of finished cement; 50 percent of this power is required for grinding. Concentric compartmentation

will result in a power saving of 20-25 percent, or 10 kwh per metric ton.

In an actual case in 1940, a shaft kiln clinker with 20 percent glass content (*i.e.*, hard to grind) required a power consumption of only 22 kwh per metric ton of cement. On a German 4900 mesh sieve, only 7 percent of the clinker remained. Hourly production was 17.5 metric tons, or 175 lbs per kw.

The main disadvantages of compartmentation are greater dead weight, some increase in dusting, and increased abrasion of the compartment walls. These disadvantages can be overcome and are less important than the power saving.

Tests to determine the crack formation and strength of reinforced concrete slabs with various reinforcing steels under gradually increasing loads

Otto Graf and Kurtz Walz, *Deutsches Ausschuss Stahlbeton*, No. 101, 1948, pp. 1-38
Applied Mechanics Reviews Apr. 1950 (Reiner)

Steel rods of different cross-sectional shapes with and without end hooks were used as reinforcement to concrete slabs. It was found that there was no advantage in using rods of noncircular cross section, but that a rough surface of the rods is of advantage. Hooks do not add to either ultimate strength or safety against cracking. When bentup bars are provided, the slabs fail through excessive extension of the steel in the tensile zone. When the reinforcement is straight, the slabs fail through the appearance of sloped tension cracks in regions between the point of application of load and the support, accompanied by slippage of steel, with steel stresses below those when bars are bent up. Bending up bars is economical.

Preliminary testing of an arch dam for water tightness and internal stresses (Barrage de Castillon. Observations et mesures lors des operations de mise a l'eau)

P. Ehrmann, *La Technique Moderne-Construction* (Paris), V. 5, No. 1, Jan. 1950, pp. 3-8
Reviewed by Alexander M. Turitzin

The author describes tests and observations made on an arch dam 100 m high, 16.8 m wide at the base and 4 m wide at the top. The dam was filled to two-thirds of its capacity and the observations were made as follows:

1. The construction joints, the rock adjacent to the concrete and the rock formations in the vicinity of the dam were examined for water tightness.
2. Possible movement of rock at the side of the dam, caused by water pressure exerted on the arch, was ascertained by extensometers located in the rock.
3. Internal stresses in the body of the dam were determined and compared with the design stresses.

The internal stresses were determined by comparing the vibrations of a cord set in a brass tube and embedded in the concrete with a calibrated standard cord. Since the frequency of vibration is a function of the length of the cord, the change in the length of the cord produced by a change in the concrete stress would cause a change in the frequency of vibration. The cord was vibrated by an electro-magnet and the amplified vibrations were heard at a listening post located outside the dam.

Evolution of dam contruction in Italy (L'Evolution des barrages en Italie)

M. C. Marcello, *La Technique Moderne-Construction* (Paris), V. 5, No. 2, Feb. 1950, pp. 55-58
Reviewed by Alexander M. Turitzin

In Italy there are 175 dams of all types, at least 10 m high, measured from the river bed. The gravity type of dam is most frequently used. There are 109 gravity type dams, which is 62 percent of the total number. Before 1900, only four gravity dams and two earth dams were built. Since 1900, an additional 105 gravity dams, plus 23 single-arch dams, 10 deck and multiple-arch dams on buttresses, 17 dry masonry dams, 5 hollow-gravity dams and 2 mixed type dams have been built. At the beginning of the century, the gravity type dam, which is an inheritance from the ancient Spanish dam builders, but with a more rational triangular profile, was predominant. Later, the arch type dam, more economical in material, was built whenever topographical conditions would permit, so that from 1915 to 1924, of the total of 45 dams built in Italy, 22 were of the gravity type and 10 of the single- or multiple-arch type. The failure of the Gleno multiple-arch dam in 1923 started a campaign against all types of arch dams; for the next ten years gravity type dam construction predominated. At present, the arch type has returned to its important

place in dam construction in Italy. When completed, the dam of S. Giustina on the Noce (Trentin) now in final stage of construction will be the highest single-arch dam in the world, 152 m high above the foundation.

The author advocates a hollow gravity type dam of triangular cross section. He claims a reduction in concrete of 30 percent by volume. Additional advantages over the gravity type are the elimination of uplift pressure and a better dissipation of heat during and after construction.

Tests of prestressed concrete (Beproevingen op voorgespannen beton)

G. Baab, *Cement*, No. 13-14, 1950, pp. 251-255
Reviewed by J. W. T. Van Erp

Tests were executed on one beam picked at random out of 150 beams prefabricated for a building. The beam (T-section) was stressed 24 days after pouring, according to the Blaton-Magnel system. Design was based on allowable stresses for concrete: f_{cc} = 1850 psi (compression), f_{ct} = 185 psi (tension); for steel: f_{si} = 140,000 psi (during prestressing) f_{se} = 125,000 psi (after prestressing)

Concrete mix, placed and vibrated in steel forms, was 1 cement: 1 sand: 3½ gravel. Water cement ratio 0.4 (4.7 gal. per sack of cement).

The following tests were successively made.

1. Loading was applied similar to what the beam would be subject to in the actual structure. First cracks in the tensile zone appeared after the live load was further raised. This load was increased to 2½ times and deflections were measured. Cracks disappeared immediately after removal of this load. Deflections also returned to zero but did so slowly. At maximum loading the modulus of elasticity of the concrete was found to be 4,950,000 psi.

2. In the impact test a 2600 lb weight was dropped from increasing heights. From the measured deflections it was found that the impact factor for a weight-drop of 3 ft was 2; this factor giving the equivalent static load.

3. Under normal dead and live load the underside of the beam was heated to 1580 F in about one hour. As was to be expected deflection increased due to expansion of the steel, as was proved by the complete disappearance of the extra deflection after cooling down. The steel was apparently adequately protected first by the injected cement grout in the reinforcing channel and secondly by the 2-in. concrete coverage on all sides, although this latter had disintegrated and was pulverized over a large area by the heat.

4. A final load test similar to the first except that the load was increased until failure of the beam by crumbling of the concrete in the compressive zone. Ultimate load was 3 times the total dead and live load for which the beam was designed.

The tests fully confirmed the assumptions on which the design was based, even though the tests were taken consecutively on the same beam so that the permanent damages accumulated in the latter tests.

The new Sclayn bridge (Namur), 426-ft prestressed concrete structure over the Meuse (Le nouveau pont de Sclayn (Namur), ouvrage en beton precontraint de 130 metres de portee, sur la Meuse)

La Technique des Travaux, May-June 1950, No. 5-6
Reviewed by Charles C. Zollman

What is believed to be the largest continuous prestressed concrete structure—based on the Magnel-Blaton sandwich plate—is described and illustrated.

The structure completed at Sclayn, Belgium consists of one continuous girder overbridging the Meuse River, supported on an abutment at the left river bank, an existing pile bent at mid-river, and a pile bent on the right river bank, thus giving two spans of 206 ft each. A single approach span of 52.5 ft is supported on the river bank pile bent and an abutment. The roadway is 23 ft wide and has a 5 ft cantilever sidewalk on each side. The main feature of the structure is the continuous girder made of one box-girder with two longitudinal partition walls throughout the entire length of the girder. In the three "cells" thus formed, a total of 36 wire units are placed.

Each wire unit contains 48 wires of 0.276 in. diameter which are distributed uniformly across the width of the bridge. They extend for the full 412 ft length of the girder, are straight in each span but change direction slightly over the river pile bent. Stressed to 121,000 psi a total prestressing force of

about 6600 U. S. tons is developed, resulting in a total elongation of each wire of 22 in.

Concrete was placed for the box girder from mid-June to mid-August, 1949. The prestressing was performed at the end of September. Static and dynamic loading tests were made January 5, 7, and 19, with the official opening of the bridge February 9, 1950.

The use of fly ash as structural material

K. Korth, *Bauplannung Bautechnik*, V. 3, Apr. 1949, pp. 125-132

Applied Mechanics Reviews Mar. 1950 (Winterkorn)

Brown-coal fly ashes when mixed with water harden in air due to a combination of hydraulic cementing action, gypsum-like binding, and lime-like carbonization. The hardenability and relative participation of the three described mechanisms depend upon chemical composition, fineness, and degree of fusion. The tentative German norm DIN 4209 established two quality classes, Br 40 and Br 10, giving respective 28-day cube compression strengths of 40 and 10 kg per sq cm.

In 1947 the magistrate of Greater Berlin suggested use of this cement in permanent structures in (1) masonry mortars, (2) interior and exterior wall plasters and (3) manufacture of building slabs (100 x 33 x 5 cm). The author critically discusses the properties of fly-ash mortars, plasters and other compositions in comparison with those of the commonly employed standard materials for the same class of work; he concludes that the principal field of fly-ash utilization is in masonry mortars and wall plasters, and that special high-quality ashes, which can be utilized in the manufacture of structural units with higher strength and heat-insulation requirements, are available only in restricted quantities.

Concrete beams with compression reinforcement (Tryckarmerade betongbalkar)

Arne Johnson, *Betong* (Stockholm), V. 35, No. 1, 1950, pp. 113-125

The first part of the paper deals with various methods used in the design of concrete beams provided with compression reinforcement. A description is given of the method stipulated in the Swedish standard specifications, the method given in the regulations of the American Concrete Institute, and the method of limit design. Furthermore, the author advances a modified method of design which may conveniently be used in those cases where the classical method or a similar method is prescribed for the design of the cross sections provided with simple reinforcement. In the case of transition from simply reinforced cross sections to those provided with double reinforcement, the method of design suggested in this paper results in equivalent structures. The compression reinforcement is correctly utilized with due regard to its quality. Moreover, the calculations are simplified. Normally, neither the Swedish specifications nor the American regulations take into account the quality of compression reinforcement. The method devised by the author is based on the principle of addition, and may be regarded as a modified method of limit design.

In the second part of the paper an account is given of a series of tests made on concrete beams with compression reinforcement. The maximum ratio of reinforcement used in the tests was 2.5 percent. The agreement between the values of the ultimate moments observed in the tests and calculated by the method of limit design was good. The magnitude of the ultimate loads was found to be independent of whether the beams were reinforced with stirrups or not. Buckling of compression reinforcement took place in both cases after the ultimate load had been reached and had slightly decreased. But after buckling of compression reinforcement, the load-bearing capacity of the beams without stirrups decreased more rapidly than that of the beams provided with stirrups. Consequently, the total carrying capacity of the beams is smaller in the absence of the stirrups, and this can be of importance in structures likely to be subjected to dynamic forces.

All reinforced concrete freight depots (Les Halles a Marchandises de la S.N.C.F. a Lyon-Guillotiere)

M. Gasc, *La Technique Moderne-Construction* (Paris), V. 5, No. 1, Jan. 1950, pp. 9-17

Reviewed by Alexander M. Turitzin

The author describes the structural features of two freight depots designed to handle 50,000 parcels each day. The receiving depot is 250 m long by 60 m wide, and the loading depot is 267.3 m long by 77.6 m wide with

facilities for loading simultaneously 175 railroad cars standing on seven tracks.

In the longitudinal direction, each building consists of a series of 21 reinforced concrete sheds, each 12 m wide and supported by continuous reinforced concrete trusses resting on two intermediate and two end columns. The roof of the shed is a concrete shell 7 cm thick stiffened by inclined trusses located every 4 m and supported in their upper ends on the top chord of the front continuous truss and in their lower ends on the bottom chord of the rear continuous truss.

Contrary to general practice in the design of industrial buildings, the continuous roof bearing trusses were supported at their ends not by square or rectangular columns but by triangular shaped columns. Between these truss bearing columns running all along the sides of the building precast hollow triangular posts were erected 12 m apart. These posts run from the foundation to the underside of the roof. Except for a concrete wall 1.6 m high above grade, the building was covered with glass panels framed into the triangular posts.

The choice of triangular shaped columns was dictated by the esthetic effect produced by the shades and shadows; the sense of solidity without undue heaviness; the wind breaking effect caused by the sharp edge of the triangle and last but not least, the economical use of materials. For a conventional type building panel 2.7 m long by 15 m high, 121 cu m of concrete and 10 tons of steel reinforcing are required, while for the V-shaped column only 33 cu m of concrete and 4.3 tons of steel reinforcing were necessary. In other words, the ratio of the cubic meters of concrete to the moment of inertia was considerably decreased. It is evident, therefore, that the concrete was more economically utilized by adopting V-shaped columns.

In addition to the V-shaped columns, the semi-circular roof cornice, window frames and muntins were prefabricated. The concrete floor was placed in the early stages of construction and was utilized as a rigid support for the roof formwork. The author states that the planning of the freight depots was begun in 1946 when little structural steel was available. Under normal conditions, such as exist today, for speed of erection, structural steel would be preferred for the roof framing of this type of structure.

Tests of a continuous prestressed concrete girder (Essai d'une poutre continue en beton precontraint)

GUSTAVE MAGNEL, *Science et Technique* (Brussels), No. 3/4, 1950

Reviewed by CHARLES C. ZOLLMAN

In testing step-by-step over a period of nearly two years, from working load to cracking load and to failure load, a continuous three-span prestressed concrete girder, the author compares testing results with his computations, as the girder was designed in accordance with the theory developed in his book *Prestressed Concrete.*

The continuous I-shaped girder of constant 21 in. depth, overbridged three spans of 22 ft each. It had been designed to carry its own dead weight and a concentrated load of 6.5 U. S. tons placed either alone at mid-span of any span or at all three mid-spans simultaneously. One wire unit of thirty-two 0.197-in. wires was required, its shape was determined in accordance with the author's theory; the wires were stressed in pairs to 120,000 psi and were anchored with Magnel-Blaton sandwich plates, with losses assumed to be about 15 percent. One concentrated testing load, produced by a jack, was placed 7½ in. at each side of the centerline of the interior span, the mid-span loading being the worst condition. At the age of two months, tests were started. The first crack appeared at mid-span of the interior span at a load of 14.5 U. S. tons. The computed flexural strength was therefore in the neighborhood of 850 psi for the 7 bag cement concrete used for this girder. A flexural beam test made at the age of 20 months gave a flexural strength of 1100 psi.

Thereafter, the author "played" with the girder for about 1 year and a half, loading and unloading it suddenly over a hundred times.

The first loadings of 14.5 U. S. tons were increased to 22 U. S. tons to widen the crack which closed completely and became invisible when the loads were removed. The 22-ton load caused the ends of the girder to raise off their supports making the girders a determinate cantilever structure. Sudden emoval of the jack load caused the grirder to fall back on its outer supports with a loud thud. Having repeated this procedure

numerous times the author investigates in detail the "secondary" cracks and stresses produced thereby in the neighborhood of the interior support.

For the loading to failure the ends of the girder were prevented from raising off their supports. Failure occurred at a load of 30 U. S. tons. This failure and its implications are carefully analyzed by the author, who interprets further the observed deformations and deflections, particularly in view of the cantilever behavior of the girder during a long period of its existance. The tests indicated that the safety against the first crack is 2.07, the safety against failure 4.3 and the author concludes that the assumptions whereon his design is based are reasonable and safe.

Putting more sense into the public construction dollar

Civil Engineering, Feb. 1950, pp. 17-23
Reviewed by J. R. SHANK

Common sense can promote economy in dam construction

ROSS WHITE

This author calls attention from the construction man's standpoint to ways in which costs might be reduced by specifying less about construction procedures, leaving more discretion to the government's engineer inspector and the contractor's engineers. A good engineer on the government's side will recognize that his obligation is to get a good job done and a good way is to cooperate with the contractor's engineers. Responsible contractors cannot afford poor inexperienced engineers. A proper professional engineering attitude on both sides when neither is hampered by questionable restraints works for both high quality and economy.

Some specification details that cause increased costs are: (1) Refrigeration before and after placing. In only extreme cases is this necessary, (2) Specifications pro and con with respect to equipment systems and units therein: belt conveyors, bucket size, etc., (3) Prohibitions of aggregate types and demands for aggregate combinations with little regard to the economics of availability, (4) The demand for shallow lifts and excessive treatment of top surface of previously placed concrete, (5) Limitations on the difference in heights between adjacent monoliths, and (6) Unnecessarily rigid inspection by inspectors without engineering jurisdiction.

Simplification can save money on government buildings

FRANCIS TOMPKINS

The author gives five steps proposed to obtain economies in government building construction:

(1) Simplification and economy in structural design. He refers to 13 principles initiated in 1941 by the Citizens Housing Council of New York headed by Jacob Feld. Notwithstanding that governmental buildings are usually special monumental buildings much economy may be had through more care in the use of modular systems and similar details wherever possible. Permitting the contractor to save forms at the expense of a bit more concrete when fireproofing structural steel and using as much prefabricated concrete as possible promote economy.

(2) Modernizing and simplifying specifications. He suggests that specifications be made as brief and simple as possible avoiding restricted competition in favor of specific products, excessive number of alternates and over-emphasis on safety requirements.

(3) Over-conservation with respect to materials. Government specification writers following previous procedures fail to recognize or allow such tried and proven procedures as lightweight precast slabs, cast-in-place lightweight concrete, vacuum concrete for higher and earlier strength, cavity wall construction, and precast architectural concrete.

(4) Separate contracts. There is no more important element in economical building than efficient co-ordination of all of the work. This can be achieved only by placing full responsibility and authority in a single head such as a general contractor.

(5) Contractor-engineer relationships. A contractor who has had expensive experiences with inexperienced and unreasonable field engineers will tend to increase his bid price.

Comprehensive studies cut costs on monumental reclamation works.

GRANT BLOODGOOD

The Bureau of Reclamation takes advantage of its long-time and continued establishment to develop economy by over-all planning, research, design improvement, standardization of specifications and bidding procedures, and close cooperation with the contractors' associations. Before a particular dam is designed studies are made of the entire

river basin and the dam is then only a part of a possible later complete development. Scale models operated by highly skilled engineers determine with ample precision the final operation of the work. The use of fly ash pozzolanic material to replace 30 percent of the cement saved $4,750,000 on the Hungry Horse Dam. Sulfate-resisting cements and air entrainment are expected to save millions of dollars in the next 20 years. Standardization of designs of construction buildings and appurtenances including prefabricated houses and packaged prefabricated buildings for small substations save time and planning. Complete but simplified standard specifications with standard tolerances with which contractors may become familiar produce low bids and cost savings. Contractors are permitted to decide upon and furnish construction materials to meet the specifications. Engineers of the Bureau continually confer with such groups as the Associated General Contractors to eliminate inequitable and arbitrary provisions in the specifications and to establish confidence among contractors and suppliers in the fairness of the Bureau and so improve the efficiency of the work.

Concrete slabs reinforced in two directions (Korsarmerade betongplattor)

H. NYLANDER, *Betong* (Stockholm), V. 35, No. 1, 1950

AUTHOR'S SUMMARY

. Ultimate load and bending stiffness, pp. 29-77 (Brottsakerhet och bojstyvhet)

An account is given of experimental investigations to provide a basis for methods of designing concrete slabs reinforced in two directions. The investigations dealt with the following types of slabs: (1) square, freely supported slabs with and without reinforcement at the corners; (2) slabs clamped along two edges and differing in arrangement of reinforcement; and (3) slabs clamped along two edges and submitted to long-time loads.

Check tests were made for determining the strength and the elastic properties of concrete and steel. The principal results are summarized as follows.

1. The membrane effect, produced by the large deformations shortly before failure, caused a considerable increase (from 10 to 15 percent) in the ultimate load of the freely supported slabs. Consequently, if the design is based on the "theory of lines of fracture" (K. W. Johansen), the factor of safety referred to the ultimate load of slabs provided with two-way reinforcement will be greater than in the case of slabs reinforced in one direction.

The moments $\frac{qb^2}{20}$ for the examined slab without corner reinforcement) and $\frac{qb^2}{27.4}$ (for the examined slabs with corner reinforcement) may be expected to give adequate safety against failure in the design of square slabs.

The decrease in the effective flexural rigidity due to formation of cracks in the portion of slab in tension manifests itself as soon as the load reaches the value at which the bending stress in concrete calculated on the basis of the moment resulting from the theory of elasticity is equal to the tensile strength of concrete in bending. After the marked transition from Stage I to Stage II, the greatest flexural rigidity was observed when the slab was provided with corner reinforcement. The decrease in flexural rigidity after cracking is far more pronounced than in the case of the slab strips used for the check tests. This indicates that the system no longer acts as an isotropic slab. For this reason, the calculation of the flexural rigidity after cracking meets with difficulties. This calculation cannot be carried out unless both the torsional rigidity and the flexural rigidity at various twisting and bending moments is known, and this is not the case. However, after the cracking load had been exceeded, the flexural rigidity of all types of slabs subjected to the tests decreased to such an extent that the deformations rapidly became so large as to be considered unallowable in those structures where the deformations must remain small.

2. The types of slabs tested were found to be equivalent in respect to failure. The ratio of the ultimate moments observed in the tests to those computed from "theory of lines of fracture" is on an average slightly greater than the ratio of the ultimate moments obtained from the check tests to those calculated from the theory. However, the margin between the observed and the calculated ultimate moments is not so great as in the case of the freely supported slabs. This seems to indicate that the membrane

stresses in Series No. 2 are of less importance than in Series No. 1.

The effective flexural rigidities of all three slabs remain equal until the load reaches that value which produces diagonal cracks at the bottom of the slab. The formation of cracks above the supports had little influence on the flexural rigidity at that stage which preceded cracking at the bottom of the slab. After the appearance of diagonal cracks at the bottom of the slab, a great decrease in flexural rigidity was observed. The greatest flexural rigidity was exhibited by those slabs which were provided with the strongest reinforcement at the supports. The marked transition from Stage I to Stage II took place at that load at which the tensile stress in concrete calculated on the basis of the maximum positive moment resulting from the theory of elasticity was nearly equal to the tensile strength of concrete in bending.

3. Alternate loads and long-time load had no influence on the ultimate strength of the slabs. When the slabs were submitted to alternate loads and long-time loads, that load which caused cracks spreading over the whole slab was also found to be critical in respect to the increase in deflection. In the comparison with the short-time loads, this load was not reduced to any notable extent by the fact that the slab had been submitted to long-time load and alternate loads. The increase in deflection with time was particularly noticeable after the formation of the first cracks at the bottom of the slab.

The "theory of lines of fracture", when modified so as to take into account the membrane effect, has proved in all tests to provide an appropriate basis for estimating the ultimate load.

II. The effect of shrinkage (Inverkan av betongens krympning), pp. 78-112

Shrinkage of concrete exerts an important influence on the deformations and the moment distribution. The tests on vibrated concrete beams described in this paper have shown that shrinkage, when nonuniformly distributed over the depth of the beam, can produce a great effect on the deformations and the moment distribution in unfavorable cases. Excessive vibration increases the amount of nonuniformly distributed shrinkage. However, it was found in the tests that short-time vibration can also give rise to nonuniformly distributed shrinkage which is so great that it cannot be disregarded in estimating the structural action of concrete slabs.

The effect of shrinkage has been calculated on the basis of the theory of elasticity under some selected conditions at the supports. The following conclusions can be drawn from some typical examples dealt with in the calculations.

When the slab is freely supported, both the nonuniform shrinkage and the prevention of shrinkage by reinforcement cause an increase in deflection, and this increase cannot be neglected in estimating the flexural rigidity of the slab. It is particularly important to prevent nonuniform distribution of shrinkage over the cross section of the slab.

If the slab is clamped along two opposite edges, and if the behavior of the slab is assumed to be elastic, the ratio of the deflection caused by shrinkage to the deflection produced by the load is smaller than in the freely supported slab. It is to be considered, however, that the moments above the supports and the stresses set up in the cross sections at the supports on account of shrinkage (particularly nonuniform shrinkage) are of the same order of magnitude as, and sometimes even greater than, the corresponding moments and stresses due to the maximum load. Consequently, crack formation above the supports is facilitated, with the result that the flexural rigidity of the slab decreases in the vicinity of the supports, and the deflections become still larger. In unfavorable cases, when the reinforcement at the supports is inadequate, the stresses can probably reach the yield point. In this way, shrinkage contributes to the breakdown of the effective rigidity of the clamped slab.

The behavior of slabs clamped along one edge or along two continuous edges is similar to that of slabs clamped along two opposite edges.

When the slab is clamped along all four edges, the effect of shrinkage is less dangerous than in the above cases. The moment above the supports and the stresses in the cross sections at the supports are slightly increased by nonuniform shrinkage, but the stresses due to the prevention of shrinkage by reinforcement are hardly increased at all.